Passive Direction of Arrival (DOA) Tracking

by

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Declaration

This is to certify that

(i) the thesis comprises my original work towards the degree of Master of Technology in Information and Communication Technology at DA-IICT and has not been submitted elsewhere for a degree,

(ii) due acknowledgement has been made in the text to all other material used.

Signature of Student

Certificate

This is to certify that the thesis work entitled **Passive Direction Of Arrival (DOA) Tracking** has been carried out by **Hemant swarnkar (200411006)** for the degree of Master of Technology in Information and Communication Technology at this Institute under my/our supervision.

Thesis Supervisor(s)

Abstract

The work reported in this thesis is concerned with the Passive Direction Of Arrival (DOA) tracking problem. Traditionally model based approach is used to solve tracking problems. The measurements (DOA) are nonlinearly related to position (element of state model) and does not include any range information. This makes the tracking problem very complex. Since we also have to deal with the effects of noise and sensor uncertainties, this results in a potentially unobservable nonlinear estimation problem.

Estimation of position of target from DOA measurements which has nonlinear relation with state vector, demands the use of a linearized model approach.

The whole thesis work has been divided into two part. First part analyzes the observability problems of DOA tracking i.e. it analyze the given scenario for the unique solution and suggest some correction to get proper system parameter estimation. This part also tries to find the principle causes of filter divergence.

In second part, the thesis is concentrated on the estimation methods used to track target. In this part thesis analyzes two different estimation algorithms named Extended Kalman Filter(EKF) and Pseudo Linear Estimator(PLE).

Thesis proposed a Triangulation Estimator which estimates the target motion parameter by using the geometry formed by the target and sensors available. Finally thesis compared the performances of the algorithms considered in it on the realistic underwater scenario.

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Contribution Made By Thesis

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2. Swarnkar H. , C. Vijay kumar, "Performance Comparison of Extended Kalman Filter and Converted Measurement Kalman Filter for localization of mobile Phone" International Conference on Embedded Systems, Mobile Communication and Computing, Bangalore, Aug 2006.(accepted)

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4. Swarnkar H., C. Vijay kumar, "Passive Direction Of Arrival (DOA) tracking" Technical paper, IEEE Transactions on Aerospace and Electronic Systems, May 2006. (under review process)

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Chapter 1

Introduction

In this thesis, passive Direction Of Arrival(DOA) target positioning and tracking application is discussed. Positioning is basically the concept of finding the target platform's kinematic state, i.e., position and velocity, whereas target tracking refers to the state of an observed object. In Fig.1.1 an DOA tracking scenario is depicted.

Estimation of current state vector which typically consists of kinematics components i.e. bearing, position, velocity, acceleration etc., with the help of measurements (Direction Of Arrival) from the target is considered as a passive tracking problem. The name passive comes from the fact that the measurement can be obtained passively i.e. without active emissions from the observers part. This is clearly an advantage in military or other covert applications since the observer need not advertise its position

Nonlinear filtering techniques are use to generalize the passive target-ranging problem because the position of the target is nonlinearly related to measurement i.e. DOA or bearings (β). The geometric interpretation of the passive tracking problem is depicted in Fig.1.1.

The observer travels on a specified course, measures passively the angle β, is corrupted by noise, and then processes these data to track the target. It is clear that the measurement equation involves nonlinearities that render the solution rather involved, compare to the target-ranging problem.

The passive tracking involves the estimation of four quantities, namely position

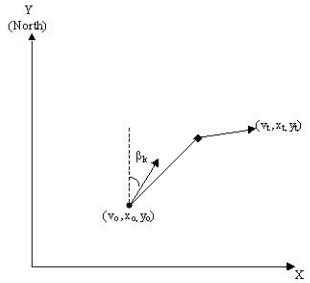

Figure 1.1: The geometric interpretation of the passive tracking problem

(x, y) and velocity (v_x, v_y). Various techniques can be found to formulate measurements that yield linear models.

The convergence i.e. tending to true value of parameters, of the various estimators depends heavily on the observer to target range and on target velocity. High target velocity and small range result in high probability of filter convergence; this is reasonable in view of the fact that in this case the bearing rate is high, resulting in the observer acquiring more information. For small target velocity and large range the probability of filter convergence is small.

1.1 Mathematical Model For DOA Tracking

The modeling of the whole tracking process can be done on the basis of a state model representation, which combines system state model and measurement model. The system state model defines the target motion, which can be used to predict the target position at particular instant of time. This prediction can be corrected or verified by the measurement model.

1.1.1 System State Model

Consider the target and observer depicted in fig.1.1. The target is traveling at an unknown but constant velocity (V_t) and located at the coordinate (x_t, y_t). The observer is traveling at a known velocity (V_o), which need not to be constant, and is located at the coordinate (x_o, y_o). State vector for target and observer can be defined as

$$X_t(k) = \begin{bmatrix} x_t(k) & y_t(k) & V_{tx}(k) & V_{ty}(k) \end{bmatrix}^T, \tag{1.1}$$

$$X_o(k) = \begin{bmatrix} x_o(k) & y_o(k) & V_{ox}(k) & V_{oy}(k) \end{bmatrix}^T, \tag{1.2}$$

where $X_t(k)$ and $X_o(k)$ are state vectors of target and observer at k^{th} sample instant respectively. V_{tx}, V_{ty}, V_{ox} and V_{oy} are velocity components in $\mathbf{X}$, and in $\mathbf{Y}$ direction, of target and observer respectively.

The dynamic model of the system is based on the relative motion of the target. So the term relative state vector, defined by

$$X(k) = [X_t(k) - X_o(k)] = \begin{bmatrix} x(k) & y(k) & V_x(k) & V_y(k) \end{bmatrix}^T, \tag{1.3}$$

The discrete-time state equation representation for the relative dynamics can be written as

$$X(k+1) = A(k+1,k)X(k) - U(k) + w(k) \tag{1.4}$$

Where A(k+1,k) is 4×4 state transition matrix

$$A(k+1,k) = \begin{bmatrix} 1 & 0 & t_{k+1} - t_k & 0 \\ 0 & 1 & 0 & t_{k+1} - t_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1.5}$$

Here, t_k is the time at k^{th} sample and the input $U(k)$ represents known observer maneuvers or the effect of observer's acceleration. The noise $w(k)$ is zero-mean, white, Gaussian noises with variance Q .

Which tells that the current state vector is linearly related to the previous state vector.

1.1.2 Measurement Model

Angle from the observers platform to the source is the only available measurement , referenced (clockwise-positive) to the **Y**-axis, as shown in Fig.1.1. It is given by

$$\beta_m(k) = \beta(k) + \eta(k) \tag{1.6}$$

where $\eta(k)$ is zero mean independent gaussian noise with variance R, and $\beta(k)$ is noise free bearing, which is nonlinearly related to state vector and it is given as

$$\beta(k) = \tan^{-1}\left[\frac{x(k)}{y(k)}\right] \tag{1.7}$$

The four-dimensional state equation (1.4) and the nonlinear measurement equations (1.6) and (1.7) define the passive DOA tracking (bearings-only tracking) process.

1.2 Literature Survey

DOA tracking problem is nonlinear problem because the measurement i.e. DOA is nonlinearly related to the target position [1,5,6,] to be estimated. It is quite special and a unique tracking solution is not always attainable. It is well known that we can estimate the parameters only when the system is observable i.e. for which estimator can produce unique solution.

System observability, in general, depends on the type of data available as well as target, and observer's platform geometry. Rigorous analysis of the observability of this distinctive system is necessary. Observability properties are much easier to

get in discrete time form because the advantage of the concise relationships between bearing and observability can be taken, steering clear of the much more complicated relationship between observer's platform state and observability. A simple and practical approach to observability analysis of DOA tracking is developed in [1,17] . In discrete time cases observability properties are derived directly from the concise relationships between target bearings (or DOA) and the observability matrix, instead of the much more complicated relationships between observer's platform state and the observability matrix. Some observability criteria are obtained very naturally using such an approach. Its potential in such applications as observer's platform maneuver optimization and tracking algorithm development is also very noteworthy.

Aidala V.J.[19] presents a method for parameter estimation, using the Kalman filter with appropriate initial conditions. This method, applied for DOA tracking, is theoretically analyzed [5]. Closed form expression for the state vector and its associated covariance matrix are introduced and subsequently used to demonstrate how DOA and range estimation interact to cause filter instability i.e. premature covariance collapse and divergence. Aidala V.J [5] has suggested that a substantial improvements in filter stability can be realized by employing alternative initialization i.e. range parameterizations ,and relinearization procedures [5,6]. S.K Rao [18] confirm the Pseudo Linear Estimator(PLE), relinearized estimator, does not require any initial estimate at all and at the same time offers all the features of the Kalman filter based pseudo-linear filter; namely sequential processing. It is also proved in [9] that when exact variance of noise in input measurements is not known and when the interval between successive measurements is of the order of 15 to 30 seconds with high aperiodicity PLE is a better solution.

The PLE [5,6,9,18] is biased in nature. It is shown in [7] that once observer's platform executes maneuver, only the estimated range vector remain biased and the corresponding velocity vector becomes asymptotically unbiased. Further investigation reveals that this range bias is highly dependent upon geometry and can be altered by additional observer's platform maneuver.

In [12], a new globally convergent nonlinear observer, called the Modified Gain Extended Kalman Observer (MGEKO), has been developed for a special class of systems. This observer structure forms the basis of a new stochastic filter mechanization called the Modified Gain Extended Kalman Filter (MGEKF)[12]. A sufficient condition for the estimation errors of the MGEKF to be exponentially bounded in the mean square is obtained. Where [13] gives the much simpler derivation of the modified gain function for bearings only measurement problem as defined by Song and Speyer [12]. It confirms that the MGEKF does indeed perform better than the standard EKF. K.S. Rao [14], explored MGEKF algorithm for underwater applications.

S.C. Nardone, A.G.Lindgren and K.F.Gong [8] developed Maximum Likelihood Estimator (MLE) for passive target tracking using batch processing. K.S. Rao in [15,21] converted this batch processing into sequential processing so that it is useful for real time application using bearings only measurements. Adaptively, the weight of each measurement is computed in terms of its variance and is used along with the measurement, making the estimate a generalized one.

In [5], it is shown that the Cartesian coordinate EKF exhibits unstable behavior characteristics i.e. divergence of the filter. Filter get diverge because of dominant effect of unobservable part of error covariance. In contrast, Aidala V.J.and Hammel S.E [10], formulate the estimation problem in Modified Polar (MP) coordinates leads to an EKF which is both stable and asymptotically unbiased without imposing any restrictions on observer's platform motion, thus, prediction accuracy inherent in the traditional cartesian formulation is completely preserved. In addition, these equations reveal that MP coordinates are well suited for bearings-only because they automatically decouple observable and unobservable components of the estimated state vector. Such decoupling is shown to prevent covariance matrix ill-conditioning Which is the primary cause of filter instability. Further investigation also confirms that the MP state estimates are asymptotically unbiased. A comparison between the recursive and iterative batch algorithms for MP coordinates also presented in [11]. The Analysis in this paper, reveals that the primary difference between these two estimation schemes

is their linearization procedure.

1.3 Organization Of Thesis

The thesis is organized into two parts. The dynamic system analysis is carried out in first part of the thesis. This part investigates the principle causes of system instability during the estimation process. In same section, thesis also entitled the problem of observability. Analysis of observability investigate the relation between the system observability to the measurements(DOA) for different cases.

There are many methods have been proposed for DOA tracking problem [5,9,10,11,12,15,23,31]. This thesis considers only two of them; Extended Kalman Filter(EKF) and Pseudo Linear Estimator(PLE). Thesis gives the deep analysis as well as the simulation results of these two estimator

It uses triangulation Estimator for target tracking in the underwater scenario. Triangulation Estimator takes the advantages of the geometry formed by the target and observer's sensors. Some algebraic manipulation enables us to estimate the relative range between target and observer's platform. This new range information utilized with the measurements (DOA) to estimate the target motion parameter. Further details are given in chapter 3 in section 3.3.1.

Thesis concluded with the simulation results for the realistic underwater scenario.

Chapter 2

Target Motion Analysis Problem

Target motion analysis (TMA) is a method of tracking a submarine by using information obtained by passive means. This chapter presents single-ship TMA procedures.Silent search sonar information usually consists of an indication of the contact's bearing and, sometimes, clues to its classification. Several methods have been developed that rely on target bearing information to obtain the contact's range, course, and speed. The process of calculating these values is called target motion analysis.

Thesis work is devoted to check the performances of the estimation algorithms for underwater scenario i.e. for sonar system. There are two types of sonar sensors available to generate DOA measurements. one is HMA (Hull Mounted Array) sonar which are mounted on the body of ship. Other is Towed array sonar which are tied behind the ship as a tow. HMA sonar works for close ranges, where as towed array works for long ranges also[25].

It is well known, if an object is moving with the constant velocity, it always follow the motion equation which can be represented into state space dynamic equation (1.4) in cartesian coordinates. As in DOA tracking the measurement(DOA) is nonlinearly related to the target coordinates can be given by equation (1.6) and (1.7).

An algorithm for estimating parameter is applicable in real time only when it estimates the parameter recursively with measurement. There are two basic estimation techniques available for parameter estimation. One is Minimum Mean Square Error

(MMSE) estimator and other is Least Square Estimator (LSE).

MMSE estimation of one random vector in terms of another random vector (measurement) is such that the estimation error is

- Unbiased i.e. expected value (mean) of error should be zero
- error must be uncorrelated from the measurements

These two properties imply the error is orthogonal to measurements. We can say the MMSE estimator is overall best if random variable (measurement) are gaussian [1].

LSE based on a set (or sequence) of linear measurements of an unknown constant parameter is a linear function of stacked measurement vector. This is the batch form of the LSE

LSE can be rewritten in recursive form, the estimate is based on a given set of measurements. The current estimate is a linear combination of

- The previous estimate and
- Last measurement.

2.1 Model Based Estimation

In recent years, the state model based filter(Kalman filter) has been utilized extensively for passive target motion analysis (TMA). Kalman filter is linear, model based, stochastic, recursive, weighted least square estimator. Kalman filtering is a technique for estimating the state of a dynamical system from a sequence of incomplete or noisy measurements. The measurements need not be of the state variables themselves, but must be related to the state variables through a linearizable functional relationship.

The flowchart of one cycle of Kalman filter[1] is presented in fig. 2.1. At every stage k the entire past is summarized by $X(k|k)$ and the associated covariance $P(k|k)$.

The state cycle consist of

- state and measurement prediction (time update)
- state update (measurement update)

The state update requires the filter gain i.e. weighting factor, obtained in the course of the error covariance calculation. The error covariance calculations are independent of the state and can, therefore, be performed offline

The flowchart given in Fig.2.1 is defined only for discrete time linear system.

Unfortunately, in DOA tracking applications, measurement model is a nonlinear function of state vector. Since a direct application of Kalman filter is not possible. Approximate algorithms, such as the Extended Kalman Filter (EKF) [5] must be employed. For the problem under consideration, this particular estimation scheme takes the form

$$X(0|0) = initial\ estimate\ of\ state\ vector. \tag{2.1}$$

$$P(0|0) = initial\ estimate\ of\ state\ vector\ error\ covariancematrix. \tag{2.2}$$

$$X(k|k-1) = A(k,k-1)X(k-1|k-1) - W(k) \tag{2.3}$$

$$P(k|k-1) = A(k,k-1)P(k-1|k-1)A^T(k,k-1) \tag{2.4}$$

$$H(k) = \partial h \partial X|_{X=X(k|k-1)} \tag{2.5}$$

$$G(k) = P(k|k-1)H^T(k)[H(k)P(k|k-1)H^T(k) + \sigma(k)^2]^{-1} \tag{2.6}$$

$$X(k|k) = X(k|k-1) + G(k)[\beta(k) - h[X(k|k-1)] \tag{2.7}$$

$$P(k|k) = [I - G(k)H(k)]P(k|k-1),\ k = 1,2,3,\ldots \tag{2.8}$$

where $A(k,k-1)$ is state transition matrix given in equation(1.5) and G(k) is Kalman gain at K^{th} time instant. The symbol "T" denotes matrix transposition. Here, $X(i|j)$ and $P(i|j)$ for i,j =1,2,3,... denote estimation of the true state vector $X(i)$ and its associated error covariance matrix $P(i)$, respectively, based on j data measurements.

Although equation set (eq.2.1 - eq.5.10) is useful for numerical work, it is not

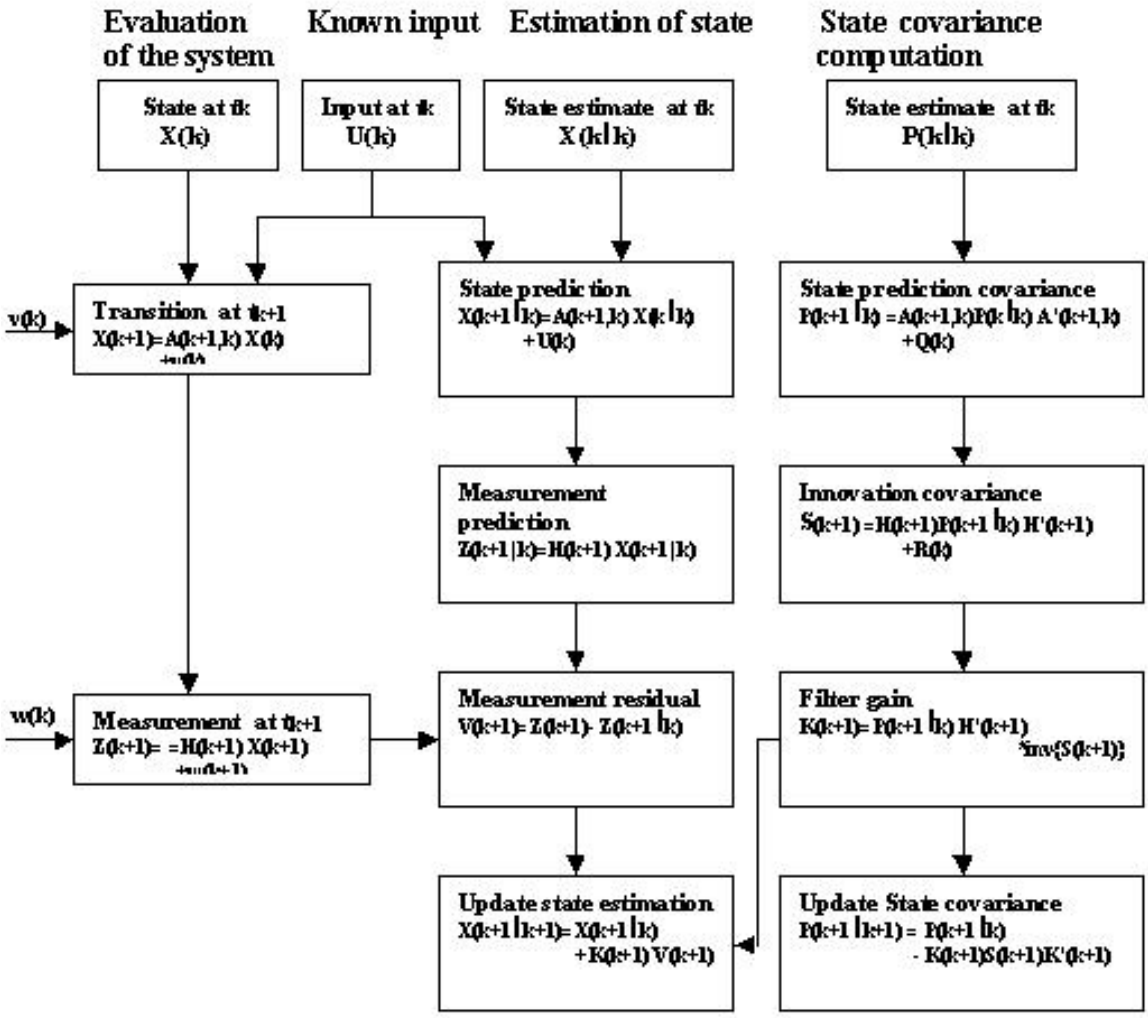

Figure 2.1: Flowchart for Kalman filter

readily amenable to analysis. For the analysis, we represent these equations in non-recursive form of the filter[19,24]. It is more convenient to work with $X(0|k)$ and $P(0|k)$. One such representation is obtained by reformulating the original dynamic estimation problem for $X(k)$ as state estimation problem for $X(0)$ given in [24]. To complete a mathematical formulation of the problem it is necessary to define a system performance index J(k),

$$J(k) = 1/2\{[X(0|k) - X(0|0)]^T P(0|0)^{-1}[X(0|k) - X(0|0)] + \sum_{l=1}^{k}[\beta(k) - h[X(k|k-1)]^T[\beta(k) - h[X(k|k-1)]\} \quad (2.9)$$

Here, $X(0|k)$ denotes the filter estimate of $X(0)$ based on the k data measurements, and $X(0|0)$ and $P(0|0)$ are a priori estimates of $X(0)$ and its associated error covariance matrix, respectively.

The filter equations for parameter can be derived by minimizing the system performance index equation (2.9). Since J(k) is a function only of one independent vector $X(0|k)$, its minimum is determined by taking differentiation of equation (2.9) with respect to $X(0|k)$, setting it to zero, and solving equation for $X(0|k)$. As given in [19], this will gives desired filter equation. By using equation (1.4) we can represent the analytical equation for Kalman filter[5], given as

$$X(k|k) = A(k,0)X(0|k) - \sum_{j=1}^{k} A(k,j)W(j),\ k = 1,2,3,\ldots \quad (2.10)$$

$$P(k|k) = A(k,0)P(0|k)A^T(k,0) \quad (2.11)$$

where

$$X(0|k) = P(0|k)P^{-1}(0|0)X(0|0)$$
$$+ P(0|k)\sum_{j=1}^{k} M^T(j)\sigma^2(j) - \{[\beta(k) - h[X(k|k-1)]\}$$
$$+ P(0|k)\sum_{j=1}^{k}\sum_{i=1}^{j} M^T(j)\sigma^2(j)M(j)A(0,i)W(j),\ k = 1, 2, 3, \ldots \quad (2.12)$$

$$P(0|k) = [P(0|0)^{-1} + \sum_{j=1}^{k} M^T(j)\sigma^2(j)M(j)]^{-1} \quad (2.13)$$

$$M(j) = H(j)A(j, 0), \quad (2.14)$$

It should be emphasized that equation(2.10)-(2.11) are mathematically equivalent to equation set(2.1)-(5.10) accordingly, they provide an exact description of the Kalman filter.

Now consider closed-form expression for $P(0|k)$ given by equation(2.13). The second term in right hand side of equation(2.13) can be written as (derivation is given in [5])

$$C(k) = \sum_{j=1}^{k} M^T(j)\sigma^2(j)M(j) = \{\Omega^{-1}(k)B(k)\}^T\{\Omega^{-1}(k)B(k)\} \quad (2.15)$$

Where C(k) is 4×4 symmetric positive semidefinite matrix. C)k) is commonly referred to as the observability matrix, because it directly form a relation of estimator's

performance to the DOA measurements.

$$\Omega(k) = diag[\Omega_{11}, \Omega_{22}, \ldots, \Omega_{kk}] \tag{2.16}$$

$$\Omega_{ii} = R(i|i-1)\sigma(i)\ estimated\ range \tag{2.17}$$

$$B(k) = \begin{bmatrix} cos\beta(1|0) & -sin\beta(1|0) & 1\ cos\beta(1|0) & -1\ sin\beta(1|0) \\ cos\beta(2|1) & -sin\beta(2|1) & 2\ cos\beta(2|1) & -2\ sin\beta(2|1) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ cos\beta(k|k-1) & -sin\beta(k|k-1) & k\ cos\beta(k|k-1) & -k\ sin\beta(k|k-1) \end{bmatrix}, \tag{2.18}$$

$$\beta(k|k-1) = tan^{-1}[R_x(k|k-1)/R_y(k|k-1)] \tag{2.19}$$

$$R(k|k-1) = \sqrt{R_x^2(k|k-1) + R_y^2(k|k-1)} \tag{2.20}$$

Note: The derivation for equation(2.38) is given in [5].

2.2 Analysis Of Covariance Divergence

Divergence occurs when the calculated error covariance becomes inconsistent with the actual error covariance. Although there are many possible causes for divergence, a common source is system modeling errors, high measurement error, linearization error.

In this section we analyze how the DOA and range estimation errors can induce the premature covariance collapse and divergency of subsequent system. Further analysis reveals that such erratic behavior is frequently precipitated by use of conventional initialization techniques.

If equations(2.13) and (2.38) are combined, it follows that

$$P(0|k) = [P(0|0)^{-1} + C(k)]^{-1} \tag{2.21}$$

or equivalently, see appendix-A1 for derivation,

$$P(0|k) = \{I - [P(0|0)^{-1} + C(k)]^{-1}C(k)\}P(0|0) \tag{2.22}$$

In practical tracking situation, accurate initialization data are not available, thus an examination of filter behavior under such circumstances is of particular interest,

case 1: Totally unreliable initialization

If the initial error covariance $P(0|0)^{-1}$ is a 4×4 null matrix. This initialization can be said totally unreliable because it reduces the equation (2.21) into the form,

$$P(0|k) \longrightarrow \tilde{P}(0|k) = C^{\sharp}(k) \tag{2.23}$$

Where matrix $C^{\sharp}(k)$ is the pseudoinverse of C(k) [6]. which is defined as,

1. When C(k) is full rank i.e. observable

$$C^{\sharp}(k) = [C^{T}(k)C(k)]^{-1}C(k)^{T} \tag{2.24}$$

2. When C(k) do not have full rank i.e. system is not observable

$$C^{\sharp}(k) = [C^{T}(k)C(k)]^{\sharp}C(k)^{T} \tag{2.25}$$

However observed from equation(2.22) that the matrix product $P(0|k)P(0|0)^{-1}$ does not vanish but rather is transformed into

$$P(0|k)P(0|0)^{-1} = I - C^{\sharp}(k)C(k) \tag{2.26}$$

Substituting equation(2.23) and (2.26) into equation(2.12) will give

$$X(0|k) \longrightarrow \tilde{X}(0|k) = [I - C^{\sharp}(k)C(k)]X(0|0) + C^{\sharp}(k)\sum_{j=1}^{k} M^{T}(j)\sigma^{2}(j) - \{[\beta(j) - h[X(j|j-1)]\} + C^{\sharp}(k)\sum_{j=1}^{k}\sum_{i=1}^{j} M^{T}(j)\sigma^{2}(j)M(j)A(0,i)W(j),\ k = 1, 2, 3, \ldots \quad (2.27)$$

Equation(2.27)and (2.23) describe the estimated state vector $\tilde{X}(0|k)$ and the error covariance matrix associated with observable components (i.e. $\tilde{P}(0|k)$) for the case of totally unreliable a priory data (i.e. $P(0|0)^{-1} = nullmatrix$) It is noted that, these result can not be extracted from the recursive filtering algorithms i.e. initial null covariance matrix makes further matrix zero. hence, utilization of the recursive equations implicitly requires that $P(0|0)$ be finite.

Case 2: Finite covariance matrix

To use recursive equations for tracking application, it is implicitly requires $P(0|0)$ to be finite. assume $P(0|0)$ is chosen in accordance with the formula,

$$P(0|0) = \sigma_o^2 I \quad (2.28)$$

where σ_o^2 is an arbitrary nonzero scaler, and I is 4×4 identity matrix. If equation(2.28) substitute into equation(2.21) we get

$$P(0|k) = P_u(0|k) + P_o(0|k) \quad (2.29)$$

where

$$P_u(0|k) = \sigma_o^2[I - C^\sharp(k)C(k)] \tag{2.30}$$

$$P_0(0|k) = [I + \sigma_o^{-2}C^\sharp(k)]^{-1}C^\sharp(k) \tag{2.31}$$

$$P_0(0|k) = C^\sharp(k) - \sigma_o^{-2}C^\sharp(k)[I + \sigma_o^2 C^\sharp(k)]^{-1}C^\sharp(k) \tag{2.32}$$

$P_u(0|k)$ and $P_o(0|k)$ are the covariance matrices associated with the unobservable and observable component of $X(0|k)$, respectively. Matrix relationship in equation(2.29) can be verified (Appendix A2) by multiplying both side of the expression with the matrix $[\sigma^{-2}I + C(ki)]$ and use the property,

$C^\sharp(k)C(k) \equiv C(k)C^\sharp(k)$

$C^\sharp(k)C(k)C^\sharp(k) \equiv C^\sharp(k)$

$C(k)C^\sharp(k)C(k) \equiv C(k)$

This representation of $P(0|k)$ is remain valid for even when the dynamic process is unobservable (i.e. prior to first observer's platform maneuver). Combine forgoing result with equation(2.12) and noting that

$$P_u(0|k)\sum_{j=1}^{k} M^T(j)\sigma^2(j) - \{[\beta(k) - h[X(k|k-1)]\} \equiv 0 \tag{2.33}$$

$$P_u(0|k)\sum_{j=1}^{k}\sum_{i=1}^{j} M^T(j)\sigma^2(j)M(j)A(0,i)W(j) \equiv 0 \tag{2.34}$$

leads to following expression for $X(0|k)$:

$$X(0|k) = \tilde{X}(0|k) + O(\sigma_o^{-2}) \tag{2.35}$$

The symbol $O(\sigma_o^2)$, designate that all terms not explicitly included in equation(2.35) are order of σ_o^2 or higher. A similar relationship can be find for $P_o(0|k)$

$$P_o(0|k) = \tilde{P}(0|k) + O(\sigma_o^{-2}) \tag{2.36}$$

The remaining term in equation(2.29) i.e. $P_u(0|k)$ is also exhibit consistent behavior as a function of σ_o^2: $P_u(0|k) \sim O(\sigma_o^2)$ which reflects the fact that errors in the unobservable components of $X(0|k)$ are directly proportional to $X(0|0)$.

Equation(2.29) is especially useful for determining the behavior characteristics of $P(0|k)$, and for identifying one of the principal causes of premature covariance collapse.

To analyze the expression observe before the first observer's maneuver i.e. $rank[B(k)] \leq 3$, and after observer's maneuver i.e. $rank[B(k)] = 4$

$$P(0|k) = \left\{ \begin{array}{ll} P_o(0|k) + P_u(0|k) & rank[B(k)] \leq 3 \\ P_o(0|k) & rank[B(k)] = 4 \end{array} \right\} \tag{2.37}$$

Note that the condition $rank[B(k)] \leq 3$ characterizes an unobservable estimation process, in this situation, bearing measurement alone do not provide sufficient information to uniquely determine $X(0|k)$ i.e. prior to the observer's maneuver $P_u(0|k)$ should behave as a non-null matrix.

If large value of σ_o is used for filter initialization, it follows from equation(2.37) that $P_u(0|k)$ should also contribute the dominant eigenvalues to $P(0|k)$ whenever $rank[B(k)] \leq 3$ i.e. estimation process get diverge.

2.3 System Observability Analysis

In control theory, Observability is a measure for how well states of a system can be inferred by the measurements. Formally, a system is said to be observable if, for any possible sequence of state and control vectors, the current state can be determined in finite time using only the measurements. Less formally, this means that we can watch our measurements and figure out what's going on inside the system. If a system is not observable, this means the current values of some of its states cannot be determined through measurements.

For linear systems in the state space representation, there is a convenient test to check if a system is observable. For a system with n states, if the rank of the following observability matrix given by equation(2.13)(derivation is given in [5])

$$C(k) = \sum_{j=1}^{k} M^T(j)\sigma^2(j)M(j) = \{\Omega^{-1}(k)B(k)\}^T\{\Omega^{-1}(k)B(k)\} \tag{2.38}$$

is equal to n, then the system is observable. In other word the observability matrix investigate the convergence dependency of the algorithm on measurement. The elements of matrix C(k) are made up directly by the measured bearings. So the properties of matrix C(k) determine the observability properties of the system. Hence the analysis of matrix C(k) is sufficient to analyze system observability. Here, we are considering only the part of observability matrix B(k) (equation(2.18)) for observability analysis. The matrix B(k) forms the direct relation to the DOA measurements.

$$B(k) = \begin{bmatrix} cos\beta(1|0) & -sin\beta(1|0) & 1\ cos\beta(1|0) & -1\ sin\beta(1|0) \\ cos\beta(2|1) & -sin\beta(2|1) & 2\ cos\beta(2|1) & -2\ sin\beta(2|1) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ cos\beta(k|k-1) & -sin\beta(k|k-1) & k\ cos\beta(k|k-1) & -k\ sin\beta(k|k-1) \end{bmatrix}, \tag{2.39}$$

CASE 1: Measurements are less than four {$\beta(1), \beta(2), \beta(3)$ }

In the case where less than four measurements are available, matrix B(k) has less than four rows. So,

$rank(B) < 4$

$rank(C) < 4$

This means matrix C is singular and the system is unobservable i.e. estimated parameters are not correct one. Case 1 implies that for a bearings-only or DOA

system to be observable, at least four bearing measurements are needed.

There is a traditional method, known as 4-bearing tracking, which uses only four bearing measurements. This is a deterministic parameter approach and the bearing measurements are supposed to be free of measurement error in this case. In practice, this assumption sometimes is quite reasonable when the error is known to be small enough. Under certain circumstances, this approximation is necessary when a quick and rough solution is needed. Such a solution can be used in emergency weapon control and in initiation of more advanced recursive tracking algorithms. That is why 4-bearing tracking is still maintained in some submarine fire control systems.

In our case measurements have been taken in underwater where noise is very high. So in this situation, for best estimation of parameters, measurement must be higher than four.

CASE 2: Constant bearing tangent

The situation when the bearing tangent is constant during the entire tracking process i.e.

$\tan\beta(k) = Constant$

Note that the first and the second rows of matrix B(k) are linearly related when $tan\beta(k)$ is constant. So are the third and the fourth rows. Therefore, in this case,

$rank(C) \leq 2 < 4$ which means that the system is unobservable.

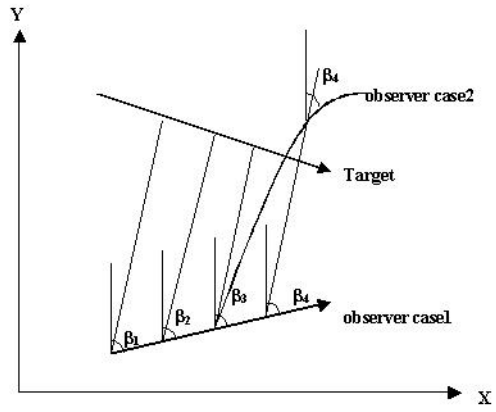

Figure 2.2: Constant bearings and constant bearing tangents

In Fig. 2.2, two different observer's motion shows the difference between the constant bearing and constant bearing tangent. Suppose all the bearing lines are in parallel in Fig. 2.2. Two observer's platform trajectory cases are shown. In case 1, the bearings stay constant and so the system is unobservable. In case 2, the value of the last bearing is different from that of the other bearings by 180^o. So the bearings are not constant, but their tangents are constant. In this case too the system is unobservable.

We note an important fact: constant bearing tangents or bearings do not necessarily mean no observer's platform maneuver. For the same reason, a bearing sequence may correspond to many observer's platform paths. So, the relationship between observer's platform states and the system observability is expected to be much more complicated than the relationship between bearings and the system observability.

CASE 3: Maneuvering of observer for measurements ≥ 4

A question of interest is the effect of the motion of the platform on the ability to estimate the target parameters. It will be shown that if the platform moves with a constant velocity in DOA tracking, then the target parameter can not be estimated

i.e. the system is unobservable.

If V_{ox} and V_{oy} denote the platform velocity components in **X** and **Y** directions respectively, then the true bearing to the target at time t_k is

$$tan^{-1}\frac{x_t(t_k)-x_o(t_k)}{y_t(t_k)-y_o(t_k)} = tan^{-1}\frac{x_t(0)+V_{xt}t_k-x_o(0)-V_{xo}t_k}{y_t(0)+V_{yt}t_k-y_o(0)-V_{yo}t_k} \tag{2.40}$$

$$= tan^{-1}\frac{x_t(0)-x_o(0)+(V_{xt}-V_{xo})t_k}{y_t(0)-y_o(0)+(V_{yt}-V_{yo})t_k} \tag{2.41}$$

$$= tan^{-1}\frac{(x_t(0)-x_o(0))\alpha+(V_{xt}-V_{xo})\alpha t_k}{(y_t(0)-y_o(0))\alpha+(V_{yt}-V_{yo})\alpha t_k} \tag{2.42}$$

for every $\alpha \neq 0$

Equation (2.40-2.42) is hold for all values of α, that is , one will obtain the same sequence of true bearings if the relative position and relative velocity are multiplied by an arbitrary constant α. Thus the infinitely many values of the target parameter vector can yield the same observation and, consequently , its parameter vector can not be estimated in this case due to above non-uniqueness [1].

Therefore, in order to estimate the target parameter vector, the platform has to have an acceleration.

Proposition: When there are no maneuvers in observer, only three bearing measurements are independent[17].

Because now both target and observer's platform travel with a constant velocity, the relative geometry between them is then determined, as shown in Fig. 2.3. V_r, c_r in Fig.2.3 denote relative velocity and relative course, respectively. $\beta(k)$ (k=1,2,3,4) are four arbitrary known bearings at four arbitrary time instants. Suppose $\beta(1)$, $\beta(2)$, and $\beta(3)$ are known, $\beta(4)$ is unknown.It

In fact, given $\beta(k)$ (k=1,2,3,4), V_r and c_r can be determined by the following equations:

$$\frac{\sin(\beta(2)-c_r)}{\sin(\beta(3)-c_r)} = \frac{(t_3-t_1)\sin(\beta(2)-\beta(1))}{(t_2-t_1)\sin(\beta(3)-\beta(1))} \tag{2.43}$$

$$V_r = \frac{R(1)\sin(\beta(2)-\beta(1))}{(t_2-t_1)\sin(\beta(2)-c_r)} \tag{2.44}$$

instants. Then $\beta(4)$ can be determined as shown in the following equation:

$$\beta(4) = tan^{-1}\frac{R(1)\sin\beta(1)+(t_4-t_1)V_r\sin c_r}{R(1)\cos\beta(1)+(t_4-t_1)V_r\cos c_r} \tag{2.45}$$

$$= tan^{-1}\frac{(t_2-t_1)\sin(\beta(2)-c_r)\sin\beta(1)+(t_4-t_1)\sin(\beta(2)-\beta(1))\sin c_r}{(t_2-t_1)\sin(\beta(2)-c_r)\cos\beta(1)+(t_4-t_1)\sin(\beta(2)-\beta(1))\cos c_r} \tag{2.46}$$

From equation(2.46) $\beta(4)$ can be uniquely determined by $c_r, \beta(1), \beta(2)$, and $\beta(3)$. Note from equation(2.43) that c_r is uniquely determined by $\beta(1), \beta(2)$, and $\beta(3)$. So $\beta(4)$ is uniquely determined by $\beta(1), \beta(2)$, and $\beta(3)$.

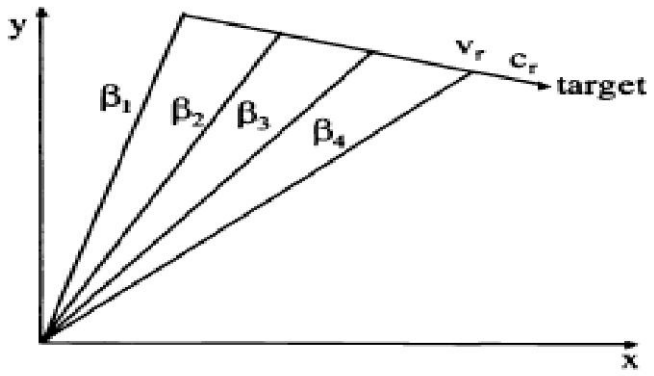

Figure 2.3: Relative geometry Target and Observer's platform

Proposition implies that if observer also maintains constant velocity during the tracking process, any bearing can be deduced by three bearings. From the viewpoint of observability, three independent bearings can sufficiently reflect the observability

information of a bearings-only tracking system without observer's platform maneuver. More bearings can not bring any new observability information. So, bearings other than the three independent ones can be defined as null bearings. If there are more than three bearings, there exists more than one choice of the three independent bearings.

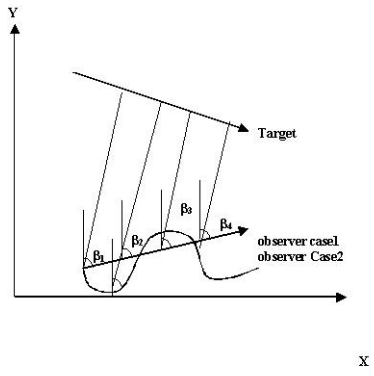

Figure 2.4: No observer's platform maneuver and null observer's platform maneuver cases

Case 4: Arbitrarily picked three bearings from a known bearing sequence

Case 4 follows naturally result of lemma. It implies that an unobservable system is not necessarily a system without observer's platform maneuver[17]. If an observer's platform maneuver leads to all null bearings except three independent ones, the system is also unobservable. Two observer's platform trajectories are shown in Fig. 2.4 . Case 1 has no observer's platform maneuver, case 2 is the maneuver case. Case 1 surely is unobservable, because no matter how many bearings are available, at most only three are independent. All the other bearings are null. As for case 2, the bearings are exactly the same as that of case 1 although with different observer's platform trajectories. There are also at most three independent bearings. So the system is also unobservable.

Chapter 3

Extended Kalman Filter

3.1 Algorithm

It is well known that the optimal estimator (Kalman filter) is only define for linear model. In DOA tracking problem, modeled in cartesian coordinate, the measurements are nonlinearly related to the state of the target. This nonlinear relation of measurement to state of target restrict us to use Kalman filter for DOA tracking.

In the extended Kalman filter (EKF) the state transition and measurement models need not be linear functions of the state but may be differentiable functions.

The equation(3.1) can be used to compute the predicted state from the previous estimate and similarly the function h can be used to compute the predicted measurement from the predicted state. However h cannot be applied to the covariance directly. Instead a matrix of partial derivatives (the Jacobian) is computed.

$$X(k+1) = A(k+1,k)X(k) - U(k) + w(k) \tag{3.1}$$

$$Z(k+1) = \beta(k) = h[X(k), v(k)] \tag{3.2}$$

At each time step the Jacobian is evaluated with current predicted states. These matrices can be used in the Kalman filter equations. This process essentially linearizes

the non-linear function around the current estimate.

The most widely used technique for linearizing a nonlinear measurements model in the form of equation(1.6) is to expand the measurement function $h[X(k)] = \tan^{-1}(\frac{x(k)}{y(k)})$ at the predicted state $X(k/k-1)$ and ignore all nonlinear terms [1,5].

$$h[X(k)] \approx h[X(k/k-1)] + \frac{\partial h}{\partial X}|_{X=X(k/k-1)}(X(k) - X(k/k-1)) \tag{3.3}$$

This amounts to approximating the nonlinear measurement equation (eq(1.6)) by linear equation,

$$Z(k) = H(k))X(k/k-1) + d(X(k/k-1)) + \nu(k) \tag{3.4}$$

where $H(k) = \frac{\partial h}{\partial X}|_{X=X(k/k-1)}$ is the jacobian of h[X(k)] given as

and $d(X(k/k-1)) = h(X(k)) - H(k)X(k/k-1)$ is innovation.

This linearized model is adequate only when $X - X(k/k-1)$ is sufficiently small [7], which can rarely be guaranteed. Since the accuracy of $X = X(k/k-1)$ relies on that of target state propagation (i.e. dynamic model) and the previous state estimate $X(k-1/k-1)$.

Brief overview of the algorithm is as follows

Let

$X(0|0)$= Initial values of state vector.

$P(0|0)$= Initial values of state vector error covariance matrix.

The state prediction equations are

$X(k|k-1) = A(k+1,k)X(k-1|k-1)$

$P(k|k-1) = A(k+1,k)P(k-1|k-1)A(k+1,k)^T + Q(k)$

$H(k) = \frac{\partial h}{\partial X}|_{X=X(k|k-1)}$

$H(k) = \begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} \end{bmatrix}_{X=X(k/k-1)}$;

where

$h_{11} = \frac{y(k/k-1)}{\sqrt{x(k/k-1)^2+y(k/k-1)^2}}$,

$h_{12} = \frac{-x(k/k-1)}{\sqrt{x(k/k-1)^2+y(k/k-1)^2}}$

$h_{13} = h_{14} = 0$,

$Z(k) = \beta_m(k)$

Kalman gain equation is

$$K(k) = P(k|k-1)H(k)^T[H(k)P(k|k-1)H(k)^T + \sigma^2(k)]^{-1}$$

State and covariance update equations are

$$X(k|k) = X(k|k-1) + K_k[Z(k) - H(k)X(k|k-1)]$$

$$P(k|k) = [I - K(k)H(k)]P(k|k-1),\ k = 1, 2, 3, \ldots$$

Specification

This thesis devoted to simulation on realistic underwater data simulation. Hence the following are the specifications carried out from the technical report from the Naval Science and Technological Laboratory (NSTL) (ref [25] p.no.8).

According to this report observer's platform always have speed higher than the 6 knots (1 knot = 0.5111m/s) with turning rate higher than the 1 deg/s. The target(Torpedo) moves with the constant speed in between 12 to 30 knots with 5 to 25 Km initial range to observer's platform. From the speed of observer and target we can roughly say that the scenario gives us 30 min of simulation time. The DOA tracking scenario is observable only when observer get maneuvered. We considered 'S' shaped observer's motion which gives a better description of the scenario's observability (NSTL report suggested [25]).

3.2 Simulation Scenario:

Brief explanation of scenario is as follows.

Target is moving at constant speed. Its motion is updated at every second. The observer's platform is also assumed moving at constant speed with 3 deg/s turn rate. Its motion is also updated at every second.

The noise in one bearing measurement is uncorrelated to that of other. If the noise measurements are correlated, we apply prewhitening process to make them uncorrelated. The noise consider in this scenario is zero mean and the bearing measurements are confided with respect to north. All the specification is taken from Ref [25].

S. No	Initial range Km	Target speed m/s	Target course deg	Obs. speed m/s	Avg.	σ^2 deg^2	Fig No.
1	20	12.5	135	3	No	0.28	3.1 -3.2

From the simulation results, it is evident that EKF is not able to track target for the given simulation scenario and same is proved by Bar-Shalom and at.al [3], this paper states that the performance of EKF is much poorer than pseudo-linear and particle filter.

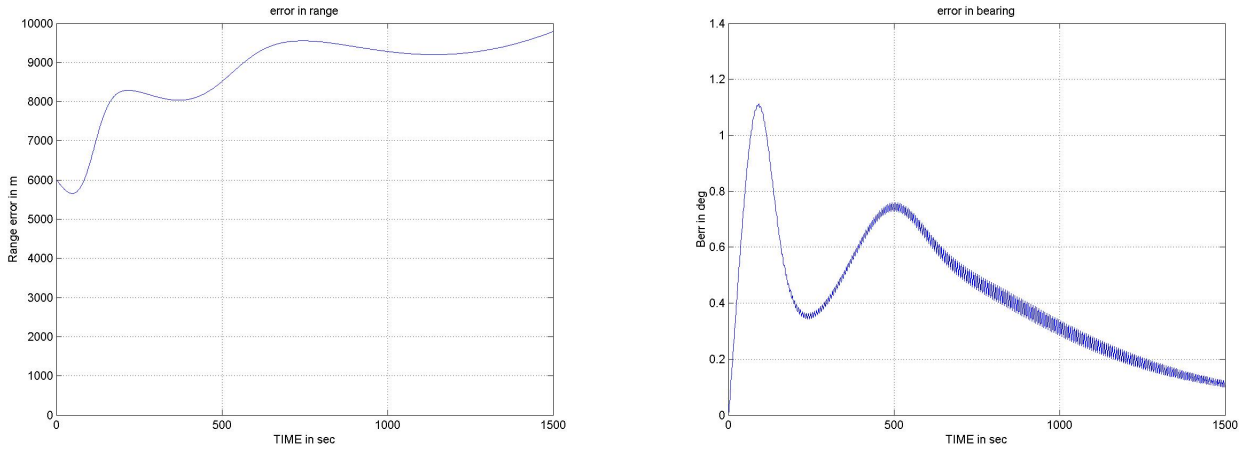

Figure 3.1: Range and Bearing error

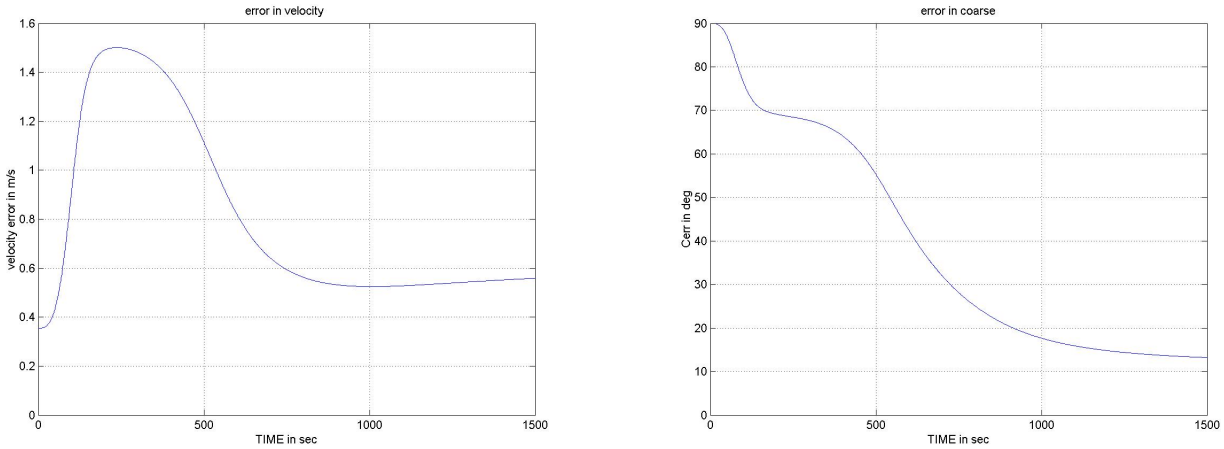

Figure 3.2: Velocity and Course error

This paper also concludes that for the passive DOA problem (like scenario considered here). EKF performance is poor and the performance of pseudo-linearized Kalman Filter and the particle filter are comparable. These conclusions motivated us to explore pseudo-linearized Kalman Filter for the scenario considered in this report. And also computational requirement of particle filter is very much higher than that of pseudo-linearized Kalman Filter.

Chapter 4

Pseudo Linear Estimator (PLE)

4.1 Algorithm

The Pseudo Linear Estimator(PLE) developed for passive target tracking using bearings-only measurements. As shown that EKF is sensitive to initial target state vector but PLE required the use of a null target state vector as an initial estimate to obtain convergence for all types of scenario. The PLE is projected in such a way that it does not require any initial estimate at all and at the same time offers all the features of the EKF based pseudo-linear filter; namely sequential processing, flexibility to adopt the variance of each measurement, etc [9].

Here the process is linearized by assuming that the range remains constant during the period of observation. Owing to this assumption, the estimator is called the pseudo linear estimator[9]. By Manipulation on original nonlinear measurements algebraically. This linear structured measurements eliminates a potential source of filter instability[5].

If bearing measurement is error free, target observer's platform geometry can be described as

$$x(k)\cos(\beta(k)) - y(k)\sin(\beta(k)) = 0 \tag{4.1}$$

In practice, however, measurement error is inevitable. In such a case, the right hand

sides of both eqs(4.1) should not be zero, say, $e(k)$

$$x(k)\cos(\beta_m(k)) - y(k)\sin(\beta_m(k)) = e(k) \tag{4.2}$$

Where $e(k) = r(k)\sin(v(k))$[5,17] is an artificial measurement error. r(k) and v(k) are predicted range and error in measurement at k^{th} instant. Now the measurement matrix become as

$$Z(k) = H(k)X(k) - e(k) \tag{4.3}$$

Where $H(k) = \begin{bmatrix} \cos(\beta_m(k)) & \sin(\beta_m(k)) & 0 & 0 \end{bmatrix}$

This pseudo measurement equation form a linear relation with the state vector, then here now we can use Kalman filter to estimate state vector.

Brief overview of the algorithm is as follows

Let

$X(0|0)$= Initial values of state vector.

$P(0|0)$= Initial values of state vector error covariance matrix. 0 The state prediction equations are

$X(k|k-1) = A(k,k-1)X(k-1|k-1) - U(k)$

$U(k)$ is deterministic observer's maneuver at k^{th} instant.

$P(k|k-1) = A(k,k-1)P(k-1|k-1)A(k,k-1)^T + Q(k-1)$

$H(k) = \begin{bmatrix} \cos(\beta_m(k)) & \sin(\beta_m(k)) & 0 & 0 \end{bmatrix}$;

Kalman gain equation is

$K(k) = P(k|k-1)H(k)^T[H(k)P(k|k-1)H(k)^T + \eta^2(k)]^{-1}$

where $\eta^2(k) = 1/2\{1 - exp[-2\sigma^2]\}$

State and covariance update equations are

$X(k|k) = X(k|k-1) - K(k)H(k)X(k|k-1)$

$P(k|k) = [I - K(k)H(k)]P(k|k-1),\ k = 1,2,3,\ldots$

Specification

The following are the specifications carried out from the technical report from the Naval Science and Technological Laboratory (NSTL) (ref [25] p.no.8).

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4.2 Simulation Scenario

Target is moving at constant speed. Its motion is updated at every second. The observer's platform is also assumed moving at constant speed with 3 deg/s turn rate. Its motion is also updated at every second.

The noise in one bearing measurement is uncorrelated to that of other. If the noise measurements are correlated, we apply prewhitening process to make them uncorrelated. The noise consider in this scenario is zero mean and the bearing measurements are confided with respect to north.

We are considering two cases of measurement processing,

1. Processing the measurement as when it comes.

2. Processing the average of 10 measurements to reduce the effect of noise. In this case assumption is made that , between these 10 measurements true bearings are constant and all noises are uncorrelated. The assumption can be validated by prewhitening process. All specifications are taken from Ref [25]

In underwater tracking, each ship has two array of sensor to find the DOA measurements. One array of sensors is mounted on the body of the ship named Hull Mounted Array(HMA) and another tied up as a two to the ship named Towed Ar-

ray(TA) sensors. It is given in the [25] that HMA gives good SNR for short range scenario and TA gives good SNR for long range scenario.Here we are considering two cases one for short range and other for long range scenario for simulation purpose,

1. If initial range is low, the bearings estimate would be generated from Hull mounted,

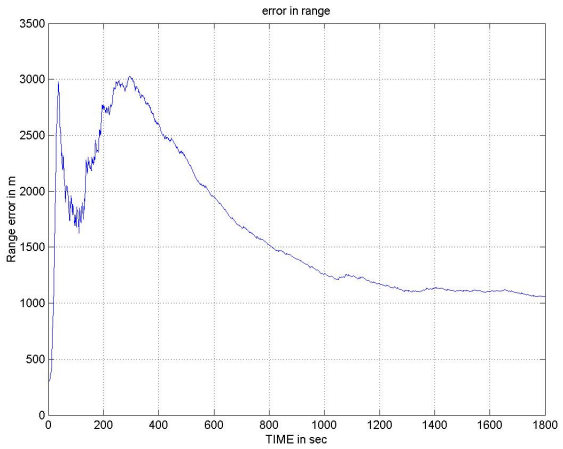
2. If initial range is very high then the bearing estimates would be generated by Towed array.

a. Hull mounted array:

S. No	Initial range Km	Target speed m/s	Target course deg	Obs. speed m/s	Avg.	σ^2 deg^2	Fig No.
1	3	15	60	1	No	0.28	
2	3	15	60	1	No	0.81	
3	3	15	60	1	No	4.256	4.1-4.2
4	3	15	60	1	10	0.28	
5	3	15	60	1	10	0.81	
6	3	15	60	1	10	4.256	4.3-4.4

b. Towed array

S. No	Initial range Km	Target speed m/s	Target course deg	Obs. speed m/s	Avg.	σ^2 deg^2	Fig. No.
1	10	5	120	3	No	0,28	
2	10	5	120	3	No	0.81	
3	10	5	120	3	No	4.256	4.5-4.6
4	10	5	120	3	10	0.28	
5	10	5	120	3	10	0.89	
6	10	5	120	3	10	4.256	4.7-4.8



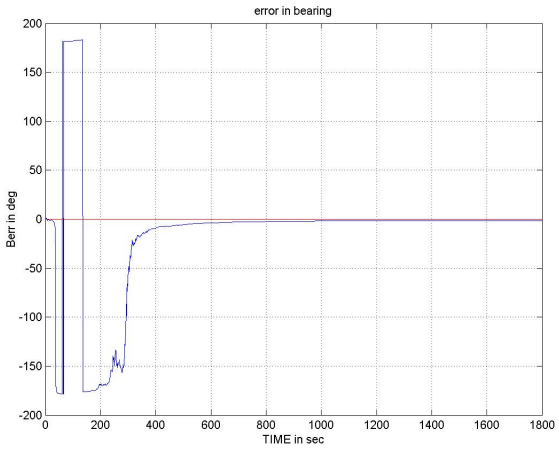
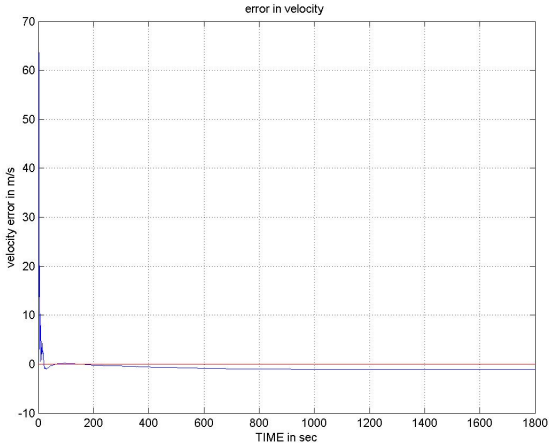


Figure 4.1: Range and Bearing error



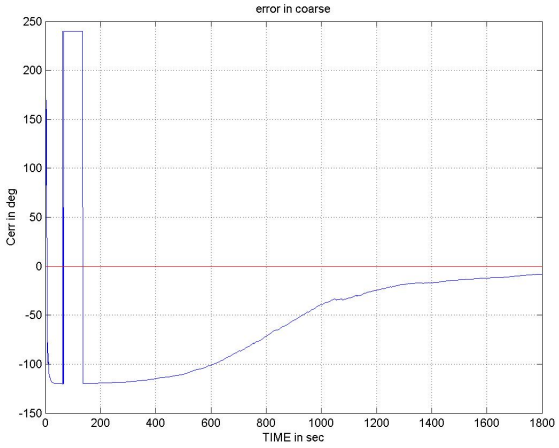


Figure 4.2: Velocity and Course error

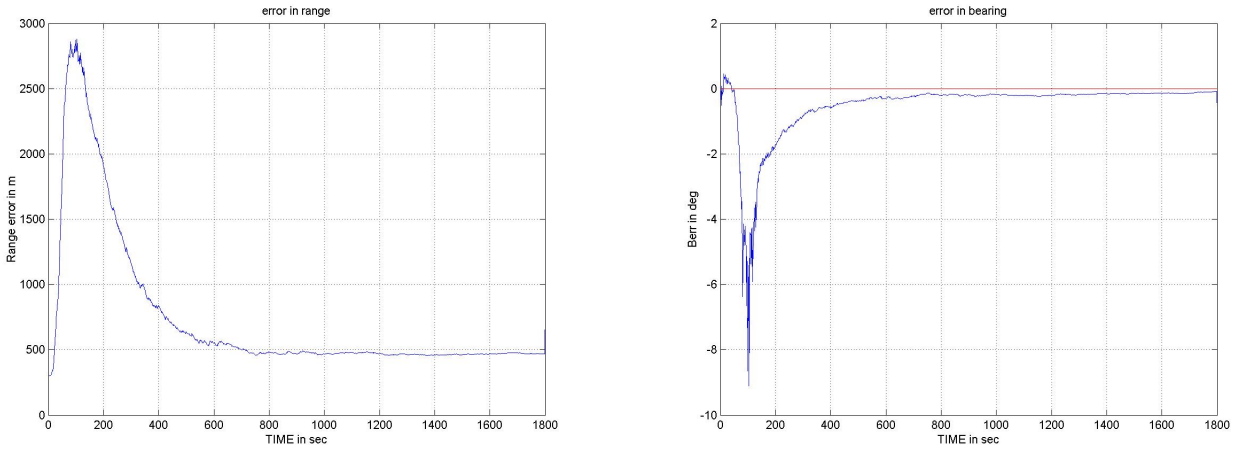


Figure 4.3: Range and Bearing Error with averaging

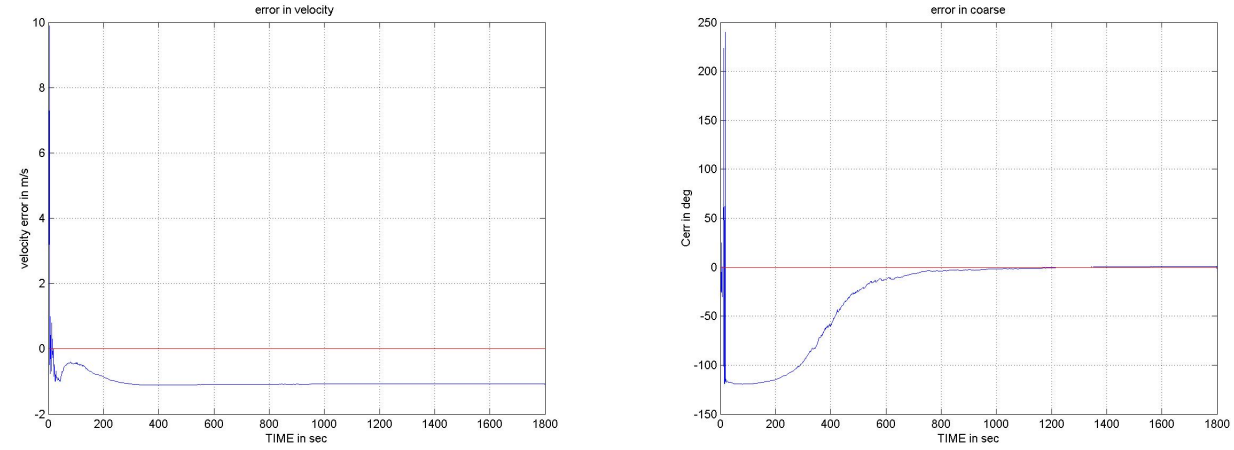


Figure 4.4: Velocity and Course Error averaging

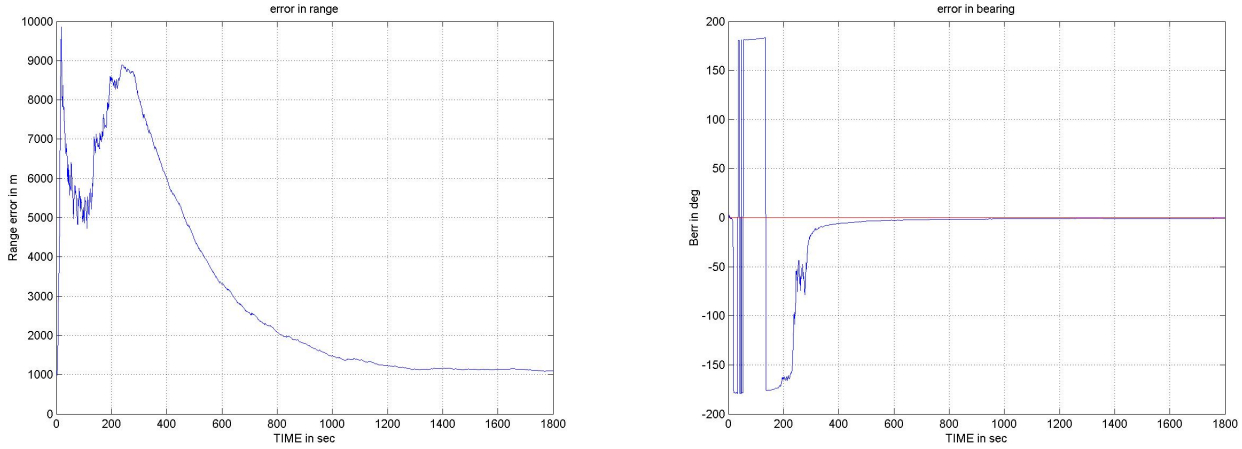


Figure 4.5: Range and Bearing error

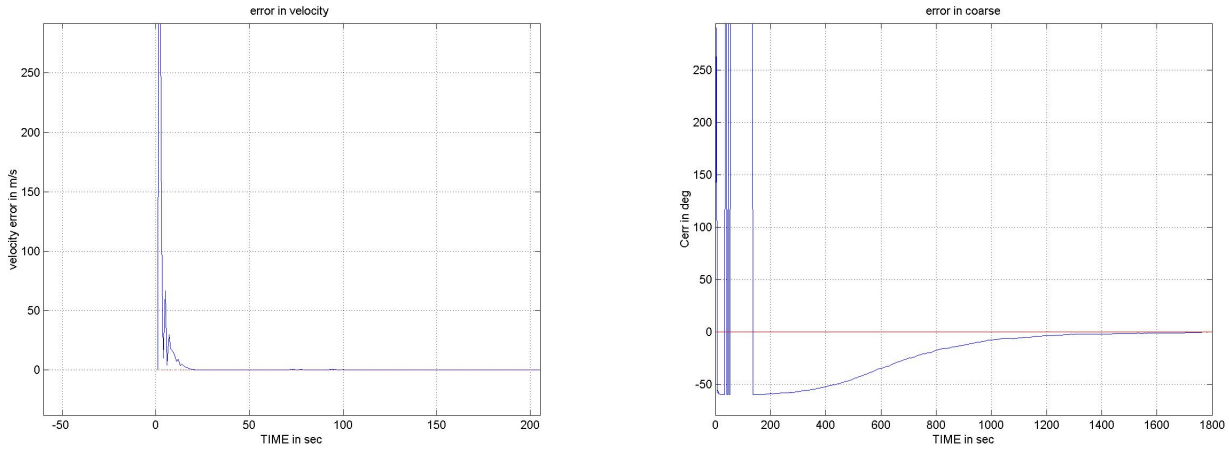


Figure 4.6: Velocity and Course error

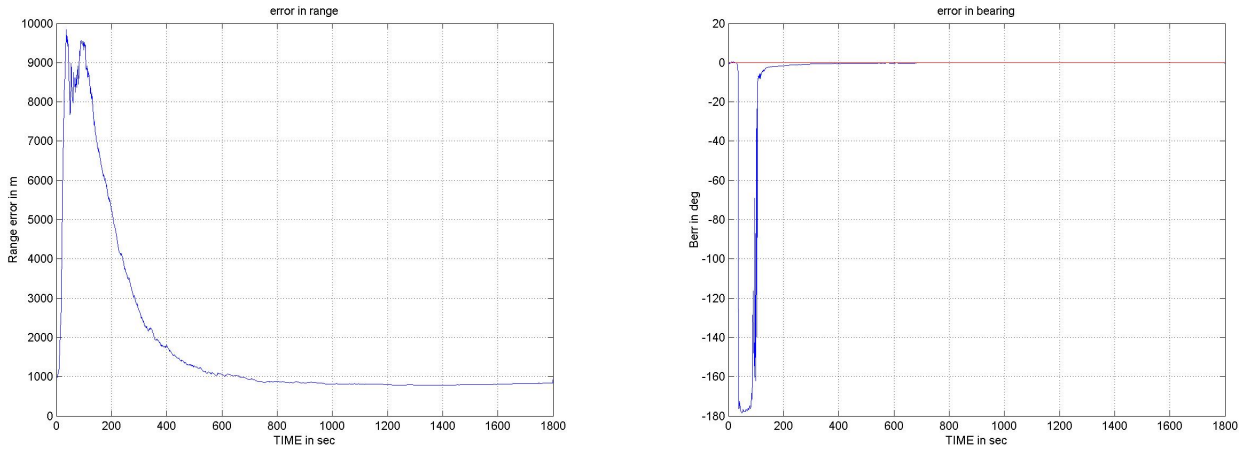


Figure 4.7: Range and Bearing error with averaging

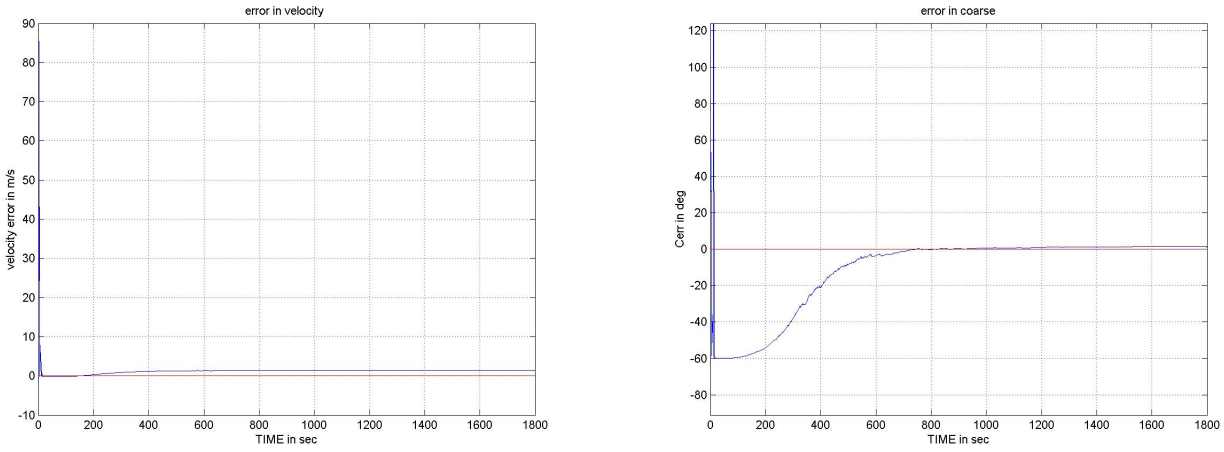


Figure 4.8: Velocity and Course error with averaging

Conclusion :

In a short range scenario averaging methodology is performing better than the non averaging and satisfying the acceptance criteria of (Ref[25],p.No.9).

In long ranges, nonaveraging perform better than the averaging method and satisfying acceptance criteria (ref[25],p.No.9).

Chapter 5

Triangulation Estimator

A geometrical approach to DOA tracking is introduced in this chapter. The approach provides a Triangulation technique to extract the range information from the measured bearings. This extracted range information is further corrected by the Converted Measurement Kalman Filter (CMKF).

In underwater tracking scenario two sensors Hull mounted and Towed arrays can give simultaneous angle measurements β_1 and β_2 from same target. Since these two sensors are separated by a minimum distance 500m, so we can estimate the range by using triangulation method.

5.1 Formation Of Triangle

Consider the scenario as defined in fig (5.1). When the target is tracked by two different sensors as shown in figure (5.1), the two angle measurements are available simultaneously at each sampling time. Therefore, the figure (5.1) is redrawn to (5.2) to define triangulation methodology. By applying sine formulae of triangle.

$$\frac{\sin(\theta 3)}{L} = \frac{\sin(\theta 2)}{R1} = \frac{\sin(\theta 1)}{R2} \tag{5.1}$$

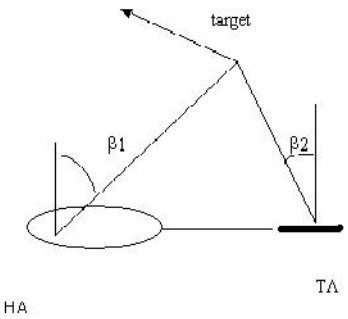


Figure 5.1: Target and sensors triangle

Here $\theta 1 = 90^o - \beta_H$ and $\theta 2 = \beta_T - 270^o$, are noise corrupted. The variance of these noises are dependent on operating SNR of the two sensors. The variances of these measurements with different operating SNRs are available in (ref[25],p.no 55 and 58)

Now range estimate is the additional information available along with bearing measurement at each sample time. This removes the primary requirement of maneuverability of the observer for tracking a constant speed target as explained in the previous section of this report, provided target does not moving along the sensors array axis Ref[29].

5.1.1 Converted Measurement Kalman filter (CMKF)

The range measurement and bearing measurements are available at each sampling time. These range and bearings measurement are transformed in the position vector by nonlinear transformation. Now these converted measurements have a linear relation with the state vector.

The nonlinear transformation is define in equation 5.2 and 5.3 than the states are estimated by using Kalman filter. This modified filter is known as Converted

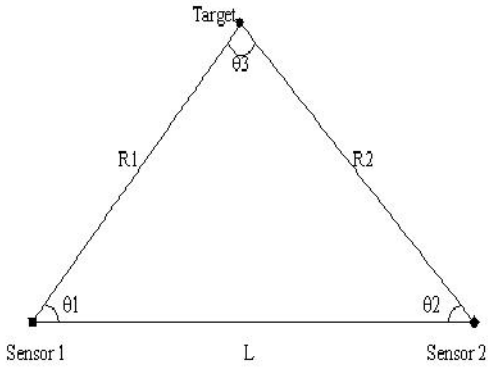


Figure 5.2: Geometric interpretation Target and sensors triangle

Measurement Kalman Filter, defined in Ref[26,28].

The brief explanation of CMKF algorithm :

The representation of range and bearing measurement into state vector's element

$$x(k) = r(k)\sin(\beta_1(k)) \tag{5.2}$$

$$y(k) = r(k)\cos(\beta_1(k)) \tag{5.3}$$

Where r(k) is measured range from target observer geometry. Now the converted measurement matrix is given as

$$Z(k) = \begin{bmatrix} x(k) \\ y(k) \end{bmatrix}, \tag{5.4}$$

Kalman filtering

Let

$X(0|0) = represent initial values of state vector.$

$P(0|0)$=represent initial values of state vector error covariance matrix.

The state prediction equation is

$$X(k|k-1) = A(k,k-1)X(k-1|k-1) - W(k) \tag{5.5}$$

Process noise covariance prediction is

$$P(k|k-1) = A(k,k-1)P(k-1|k-1)A(k,k-1)^T(k,k-1) \tag{5.6}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \tag{5.7}$$

Kalman gain equation is

$$G(k) = P(k|k-1)H^T(k)[H(k)P(k|k-1)H^T(k) + \sigma(k)^2]^{-1} \tag{5.8}$$

State and covariance update equations are

$$X(k|k) = X(k|k-1) + G(k)[Z(k) - HX(k|k-1)] \tag{5.9}$$

$$P(k|k) = [I - G(k)H]P(k|k-1),\ k = 1,2,3,\ldots \tag{5.10}$$

5.2 Simulation Scenario

Target is moving at constant speed. Its motion is updated at every second. The observer's platform is also assumed moving at constant speed in x-direction. Its motion is also updated at every second.

The noise in bearings measurement is uncorrelated. If the noise measurements are correlated, we apply prewhitening process to make them uncorrelated and the mean

value of noise is assumed to be zero. The bearing measurements is considering with respect to north. And the remaining specifications are taken from Ref [1].

First setup

S. No	Initial range Km	Target speed m/s	Target course deg	Obs. speed m/s	σ_1^2 deg^2	σ_2^2 deg^2	Fig No.
1	3	15	60	1	0.28	10.8	5.3-5.4

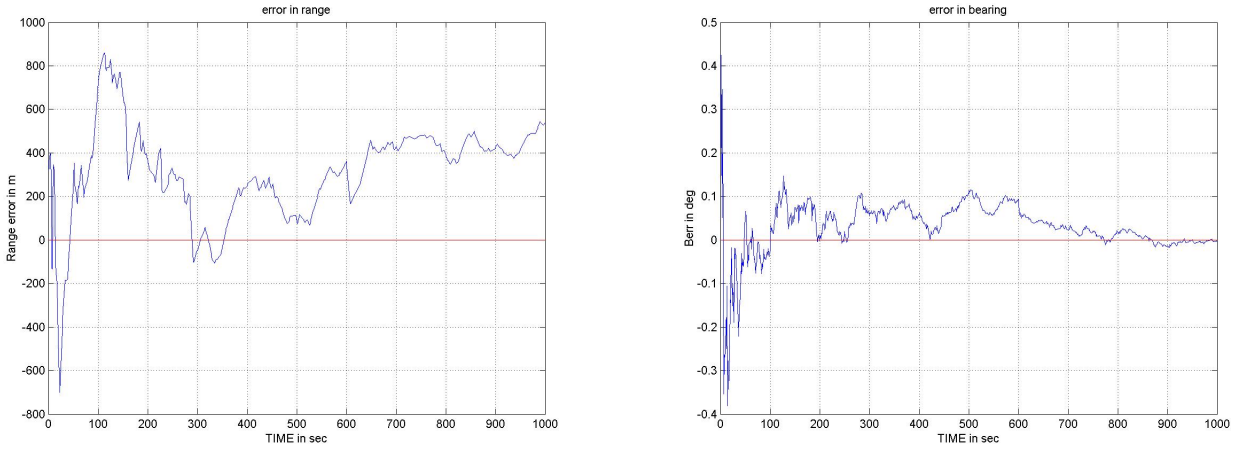


Figure 5.3: Range and Bearing error

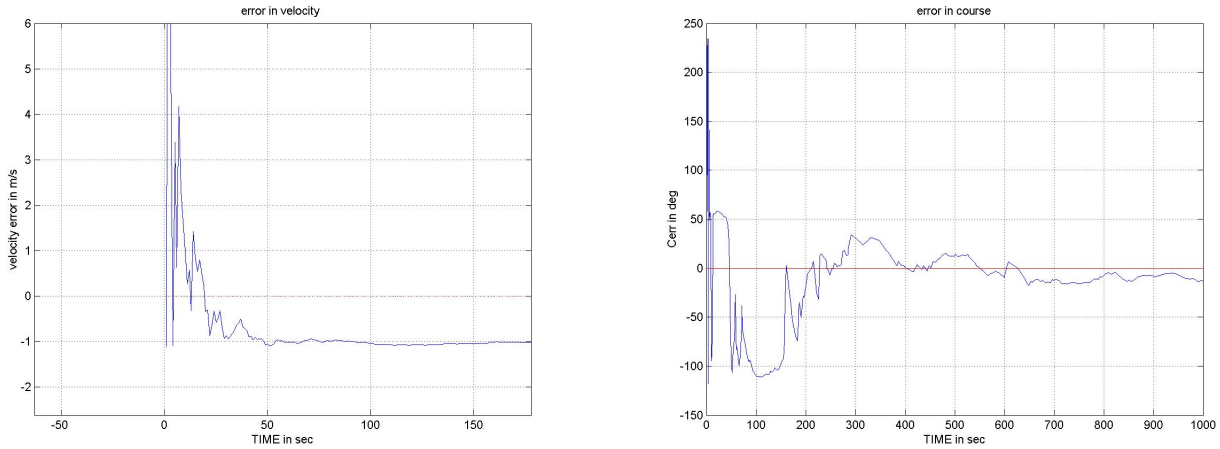


Figure 5.4: Velocity and Course error

Second setup

S. No	Initial range Km	Target speed m/s	Target course deg	Obs. speed m/s	σ_1^2 deg^2	σ_2^2 deg^2	Fig No.
1	10	5	120	3	0.28	4.256	5.5-5.6
2	10	5	120	3	0.28	0.3	5.9-5.10

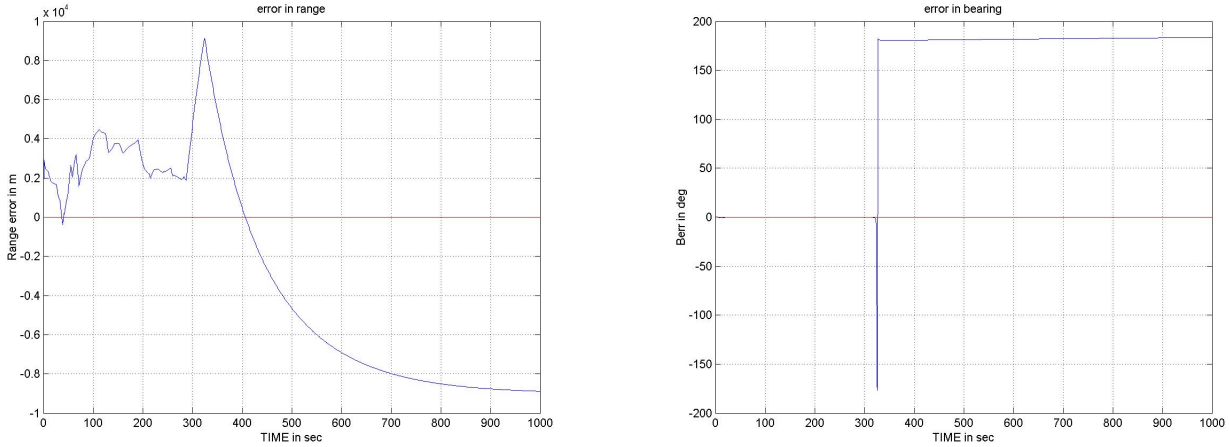


Figure 5.5: Range and Bearing error for I scenario

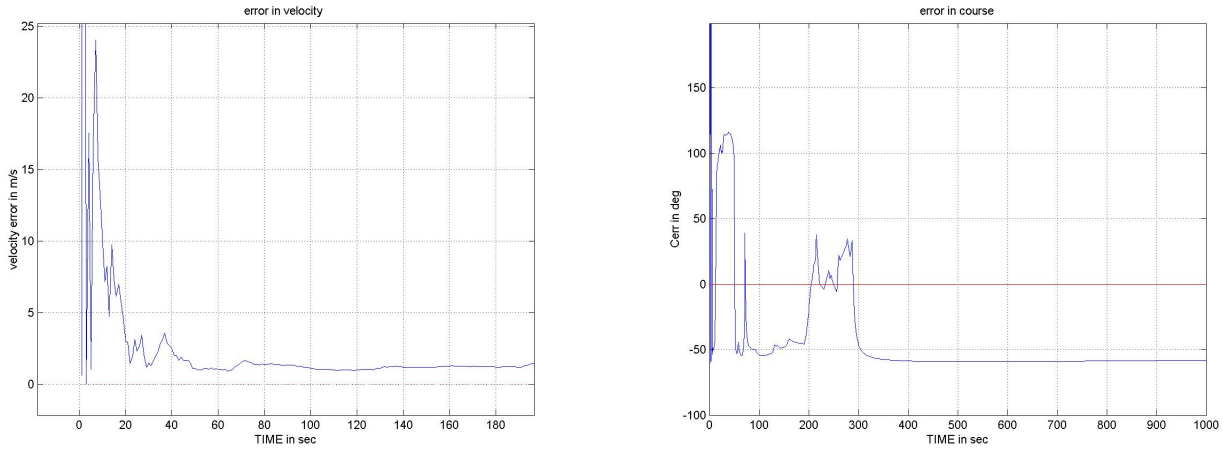


Figure 5.6: Velocity and Course error for I scenario

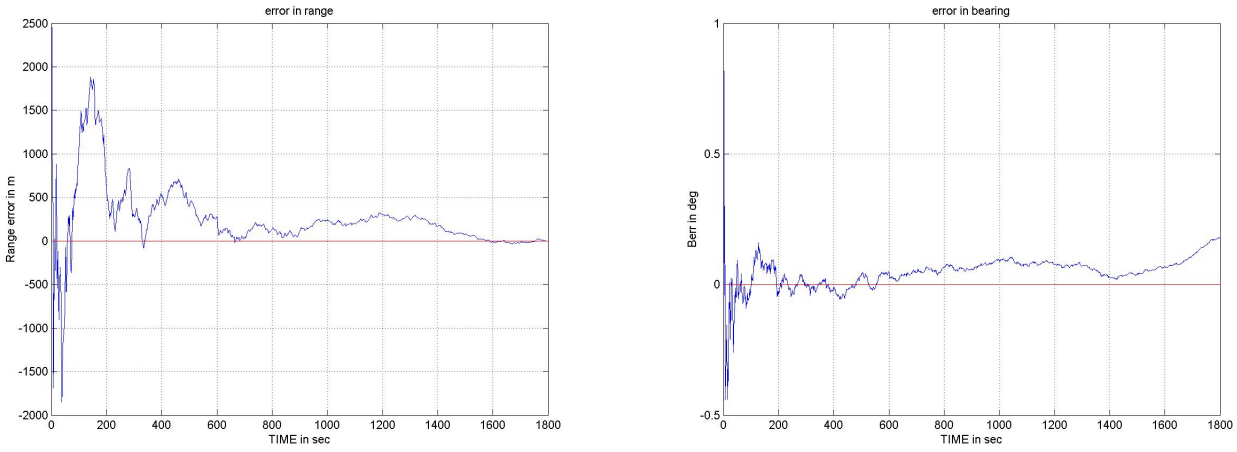


Figure 5.7: Range and Bearing error for 2 scenario

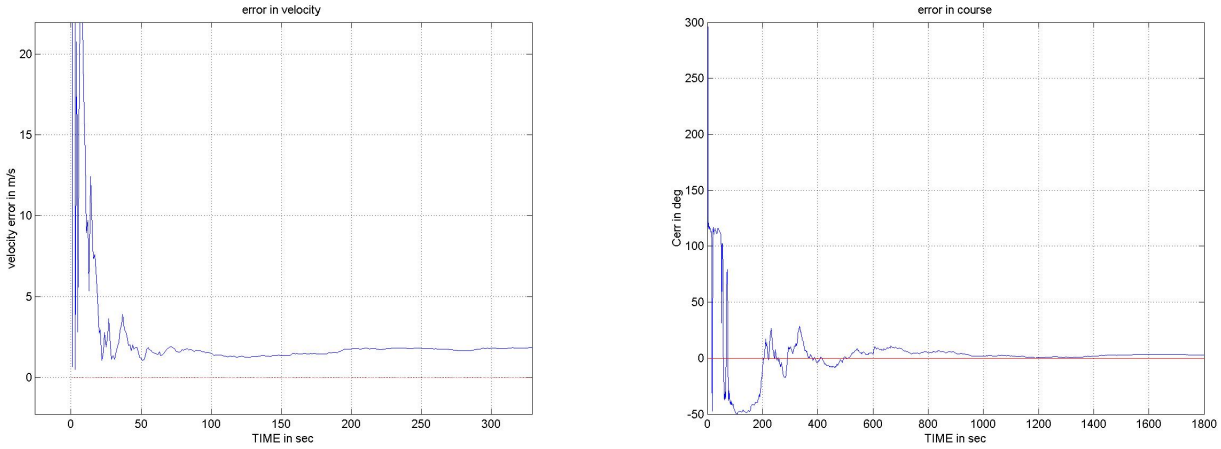


Figure 5.8: Velocity and Course error for 2 scenario

Conclusion :

In the case of low ranges CMKF based state estimation method gives acceptance performance ,as given in(Ref[25],p.No.9), with 0.28 and 10.8 deg^2 bearing variances.

In the case of long ranges this method gives acceptable performance as given in(Ref[1],p.No.9) with 0.28 and 0.3 deg^2 bearing variances.

Observation :

Assuming range error variance is limited to 30 percent of true initial range then this method gives acceptable performance even with angular variance with 10.8 and

4.256 deg^2.

Observational setup

here we are assuming, the variance of the range error is 30 percent of true range.

S. No	Initial range Km	Target speed m/s	Target course deg	Obs. speed m/s	σ_1^2 deg^2	σ_2^2 deg^2	Fig No.
1	10	5	120	3	4.256	10.8	5.9-5.10

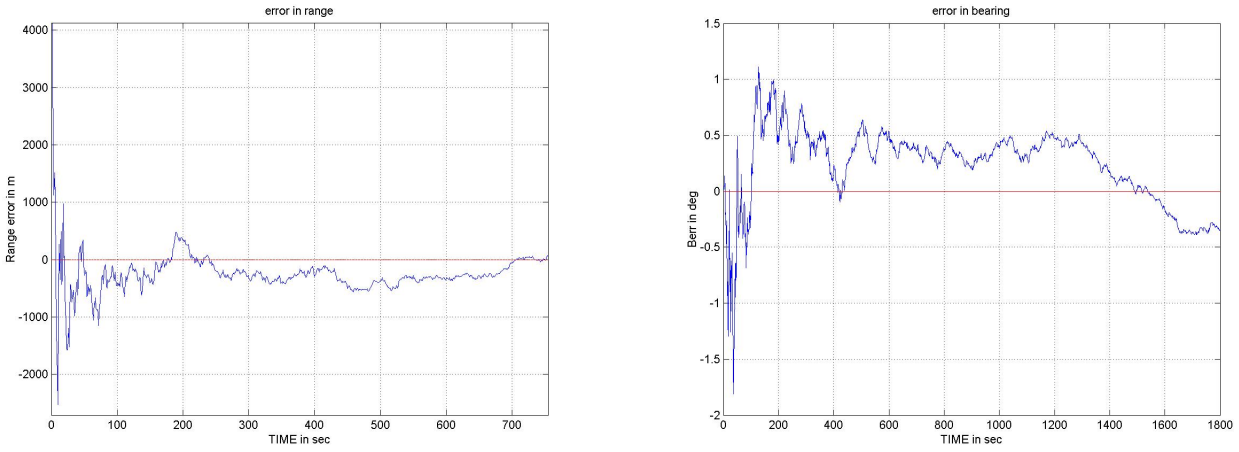


Figure 5.9: Range and Bearing error

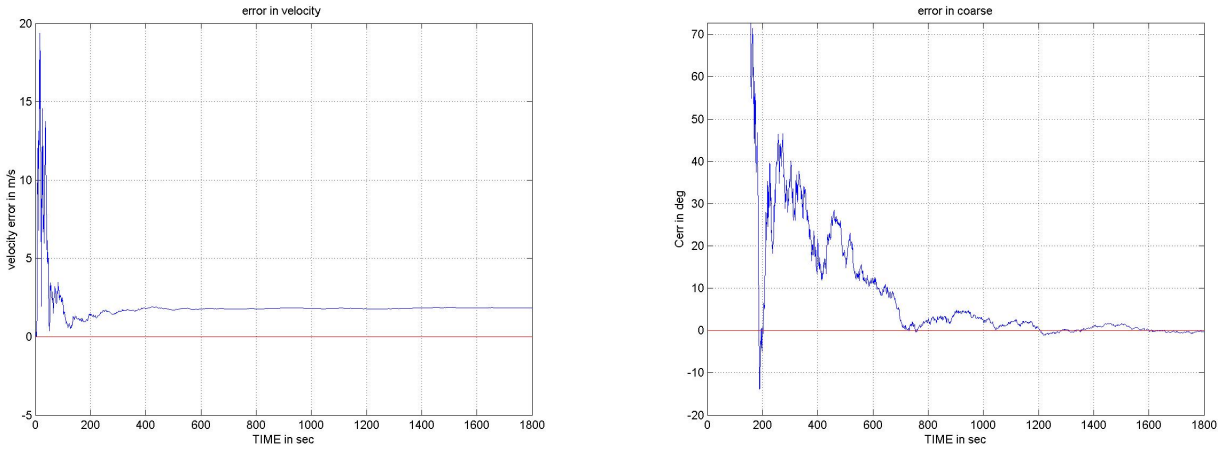


Figure 5.10: Velocity and Course error

Chapter 6

Discussion And Conclusion

It is known that the measurements (DOA) in the passive Direction Of Arrival tracking problem do not include the range information. It is also nonlinearly related to the position of the target, if we model the tracking scenario in cartesian coordinates. This makes tracking problem complex and unobservable.

Target Motion Analysis in chapter 2 has analyzed that the divergence of error covariance is highly dependent on the initial state vector; if initial error is very high then unobservable part of error covariance dominates over observable part. It is also analyzed that to get unique solution for the DOA tracking problem, observer has to maneuver in the proper way. It is also suggested that the tangent of bearings must not be constant and observer's maneuver must not follow null maneuvering case.

To check the performances of the considered estimators this thesis has taken data for realistic underwater scenario. From the simulation results in chapter 3, it has been shown that the Extended Kalman Filter (EKF) is get diverged for the given simulation scenario. The same is shown by Bar-Shalom and at.al [1] that the EKF is highly sensitive to initial estimate error. This may be the cause of divergence of EKF. We can say that EKF need highly precise data to initialize the filter to prevent divergency which is not possible in the practical case. Whereas the Pseudo Linear Estimator (PLE) gives the acceptable result shown in chapter 4.

It has proposed the Triangulation Estimator underwater scenario where we can get

two measurements simultaneously from two different sensors 500m apart from each other. In this case the observer's sensors and target forms a triangle which we can use to approximate the range by using the sine formula of triangle. This approximated range with the DOA measurement can be used in CMKF to further estimation of position and velocity of the target.

The Triangulation Estimator is very sensitive to the DOA measurement error, because small error in measurement gives high error in range approximation which may be the cause of divergence of the filter. So it has been shown that the Triangulation Estimator gives good performance for small range scenario. To get acceptable result in high DOA measurements error environment we can do an observational setup as shown in chapter 5 for CMKF.

The main advantage we gain in Triangulation Estimator is that the observer need not to maneuver for observability of the DOA tracking problem which is mandatory for other estimation algorithms.

Chapter 7

Future Extension

As it is analyzed that the filter used in DOA tracking problem is very sensitive to initialization. To solve this problem one can do the parametrization of range i.e. divide the whole range possible into the small cell and assign a filter for each cell, all filters run parallel and independently from each other. The estimated state from each filter can be weighted on the basis of the difference of the measurement and the predicted measurement.

The weighting of estimated state is very big design issue. This algorithm is also highly sensitive to the scenario where it get used. So one can take it as a challenge to design a proper weighting algorithm as well as make this method general to all types of scenario[33].

To solve DOA tracking problem one can use Particle Filter (PF) where the importance particles are generated from a particular pdf using model equations and each particle weighted on the basis of the difference of the measurement and the predicted measurements of each particle. Particle filtering is good choice when the considered problem is nonlinear and non-gaussian[3].

High computational requirement is only the drawback of Particle Filter but todays technology allowed us to bear that computational requirement. It is a big design issue to assign the weight to each particle on the basis of measurements(DOA). One can take it as a challenge to design algorithm for weighting particle.

Appendix

A: derivations

A1:

from equation (2.21)

$P(0|k)[P(0|0)^{-1} + C(k)] = I$

$P(0|k)P(0|0)^{-1} + P(0|k)C(k) = I$

$P(0|k)P(0|0)^{-1} = I - P(0|k)C(k)$

$P(0|k)P(0|0)^{-1} = I - [P(0|0)^{-1} + C(k)]^{-1}C(k)$

$P(0|k) = \{I - [P(0|0)^{-1} + C(k)]^{-1}C(k)\}P(0|0)$

A2

given equation(2.29)

$P(0|k) = P_u(0|k) + P_o(0|k)$

$P_u(0|k) = \sigma_o^2[I - C^\sharp(k)C(k)]$

$P_0(0|k) = [I + \sigma_o^{-2}C^\sharp(k)]^{-1}C^\sharp(k)$

multiply both the side by $[\sigma^{-2}I + C(ki)]$ we get,

$$P(0|k)[\sigma^{-2}I + C(ki)] = \{\sigma_o^2[I - C^\sharp(k)C(k)] + [I + \sigma_o^{-2}C^\sharp(k)]^{-1}C^\sharp(k)\}[\sigma^{-2}I + C(ki)] \tag{7.1}$$

$P(0|k)[\sigma^{-2}I+C(ki)] = \{\sigma_o^2 I-\sigma_o^2 C^\sharp(k)C(k)\}\{\sigma^{-2}I+C(ki)\}+\{I+\sigma_o^{-2}C^\sharp(k)]^{-1}C^\sharp(k)\}\{[\sigma^{-2}IC^\sharp($
$I]C(ki)$

$P(0|k)[\sigma^{-2}I + C(ki)] = \{I - C^\sharp(k)C(k) + \sigma_o^2 C(ki) - \sigma_o^2 C^\sharp(k)C(k) + C^\sharp(k)C(ki)$

$P(0|k)[\sigma^{-2}I + C(ki)] = I - C^\sharp(k)C(k) + C^\sharp(k)C(ki)$

$P(0|k)[\sigma^{-2}I + C(ki)] = I$

$P(0|k) = [\sigma^{-2}I + C(ki)]^{-1}$

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