Channel Estimation in OFDM Systems

by

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Declaration

This is to certify that

i) The thesis comprises my original work towards the degree of Master of Technology in Information and Communication Technology at DA-IICT and has not been submitted elsewhere for a degree.

ii) Due acknowledgement has been made in the text to all other material used.

Signature of Student

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Certificate

This is to certify that the thesis work entitled *Channel Estimation in OFDM system* has been carried out by *Hitesh Rathore (200811006)* for the degree of Master of Technology in Information and Communication Technology at this Institute under my supervision.

Thesis Supervisor

Prof. Manjunath V. Joshi

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An T d great!!!

To my parents and all my friends.

Abstract

Orthogonal frequency division multiplexing (OFDM) is not a new concept, but its receiver design has constant improvement. OFDM uses available spectrum efficiently and the system provides high data rates. Many standards such as Digital video broadcasting (DVB), Wireless local area network (WLAN), 3GGP LTE uses OFDM as standard. Most of the digital communication systems nowadays are wireless, where information bits are transmitted over a radio channel. These radio channels are generally multipath channel which cause intersymbol interference (ISI). To remove the effect of ISI from received signal equalization is done, which requires the channel impulse response. A good channel estimation technique can reduce the complexity of equalizer by large factor. Channel estimation can be done by sending pilot symbols at regular interval and at the receiver the channel impulse response is estimated using these received pilot symbols.

In this thesis various pilot based channel estimators are studied. We propose a new channel estimator, which is based on regularized least squares method and uses some prior information of the channel. The performance of existing channel estimators and proposed channel estimator is compared. The criteria for performance is on the basis of mean squared error (MSE) and symbol error rate (SER). Various adaptive channel estimators are also studied. We also propose a new adaptive channel estimator, based on two-dimensional normalized least mean squared (NLMS) algorithm with variable stepsize. The performance of existing adaptive channel estimators is compared with the proposed adaptive channel estimator. Performance comparison is based on MSE and the bit error rate (BER).

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Chapter 1

INTRODUCTION

1.1 Introduction

The concept of orthogonal frequency division multiplexing (OFDM) was first proposed by Robert W. Chang in 1966 [1]. It is based on multi-carrier (MC) transmission scheme [2] in which bandlimited orthogonal signals are used for multichannel data transmission. In OFDM, the signals can be transmitted simultaneously through a bandlimited channel avoiding intercarrier interference (ICI) and intersymbol interference (ISI). Implementation of transmitter and receiver for OFDM based system was not feasible for many years since it requires bank of filters at both the transmitter and the receiver. In 1971, Weinstein and Ebert proposed the method to implement OFDM using discrete Fourier transform (DFT)[3]. The implementation of OFDM systems with equalization was investigated in [4], [5] and Peled and Ruiz in [6]. In 1985, Cimini was the first to apply the OFDM in mobile wireless communications [7]. In [8], [9], Casas and Lueng discussed some experimental results on application of OFDM in mobile FM channels. Bingham [10] studied the performance and complexity of OFDM systems.

OFDM is widely used today due to its robustness to multipath effect of channel and its efficient implementation using DFT/inverse discrete Fourier transform (IDFT).

Since wireless communication continue to grow rapidly as the need for reaching data anywhere at any time rises. The requirement of high data rates also requires novel techniques for efficient transmission. However, symbol duration reduces with the increase of data rate, and dispersive fading of wireless channels causes intersymbol interference (ISI) if single carrier modulation systems are still used. To reduce the effect of ISI, in [11] it is shown that the symbol duration must be larger than the delay spread of wireless channels. As Single carrier systems are limited by inter-symbol interference (ISI) and very complex equalizers are required to reduce ISI. OFDM is a very powerful multi-carrier modulation scheme which uses large number of sub-carriers orthogonal to each other; this scheme is much effective in frequency selective channels and also supports high data rates. OFDM transforms the wideband frequency selective fading channel into large number of small narrowband channels, and can be approximated by frequency flat fading channels, which all are orthogonal to each other. This makes it more robust against large delay spread. Introduction of cyclic prefix at transmitter also reduces the complexity of receiver. The simple equalization technique at receiver requires the knowledge of channel through which it is transmitted, hence knowledge of channel statistics is necessary for simple equalization, and hence we need to estimate the channel regularly. Channel estimation is a challenging problem in wireless systems. Unlike other guided medium, the radio channels are time varying. The transmitted signal travels to receiver is affected many channel effects (reflection, scattering, multipath). Hence, corrupted signal is received at the receiver. The time varying nature of the channel limits the performance of system. Also, due to mobility of transmitter, receiver, or reflecting objects, the channel response changes rapidly. Multipath propagation, mobility, reflection, and local scattering cause the channel to spread in time and frequency. This spread depends on the time varying nature of the channel and it significantly affects the received signal. Channel estimation performance has high dependency on the channel statistics.

OFDM has been adopted in many wireless communication standards such as European digital audio broadcasting (DAB), European digital video broadcasting (DVB),

and satellite-terrestrial interactive multi service infrastructure in China. OFDM has also been considered standard for IEEE 802.11a/g/n, IEEE 802.15.3a [12], [13], and IEEE 802.16d/e [14]. The application of these standards include wireless personal area networks, wireless local area networks (WLAN), and wireless metropolitan networks. OFDM is also used as a radio transmission technique for long term evolution (LTE) of the 3rd Generation partnership project (3GPP). Various channel estimation techniques are discussed in the next section.

1.2 Background Study

In this thesis we propose new methods for channel estimation in OFDM systems. Channel estimation techniques for OFDM based systems can be grouped into two main groups: blind and non-blind. The blind channel estimation methods use statistical properties of received signal and require large number of samples of received signal. Also, these are not suitable for fast fading channels, as their performance degrades in fast fading channel [15]. On the other hand, in non-blind channel estimation methods, some portion of transmitted signals are available at the receiver, which are then used to estimate the channel. The non-blind channel estimation technique can further be divided into two main categories: pilot aided channel estimation (PACE) [16], [17], [18] and decision directed channel estimation (DDCE) [19], [20], [21]. In pilot aided channel estimation techniques, channel state information (CSI) can be estimated using training symbols known at both the transmitter and the receiver. These training symbols are also called as pilot symbols. In this method estimation accuracy depends on the density of pilots. Estimation accuracy can be increased by increasing the density of pilots, but it leads to reduction in spectral efficiency of OFDM system. Different types of pilot spacing techniques can be found in [22], [23], [24]. In DDCE method, to demodulate present OFDM symbol the channel impulse response (CIR) of previous OFDM symbol is used. The CIR for current OFDM symbol is then estimated using newly estimated symbol. Since outdated CIR is used to demodulate

present OFDM symbol, this method is less suitable in fast varying channel [25]. Pilot aided channel estimation methods are more efficient and less complex than DDCE, hence only pilot aided channel estimation methods are discussed here, which are based on block type arrangement [26]. When channel statistics are unknown and CIR is considered deterministic, then maximum likelihood (ML) channel estimator is found to be optimum [27]. It can also be shown that for AWGN channel, least squared (LS) estimation and ML estimation are the same. LS estimation has high mean squared error, and only suitable in the system where received signal to noise ratio (SNR) is high. Using channel statistics, channel estimation performance can be improved. If CSI is considered random variable then linear minimum mean squared error estimation (LMMSE) can be applied which results in better performance in terms of mean squared error, but the price paid is the computational complexity. As compared with LS estimation, LMMSE estimation has much better MSE, but it requires channel statistics and has high computational complexity. Complexity of LMMSE estimator can be reduced, if pilot symbols with constant modulus are used [28]. Using this method low rank approximation of the correlation matrix is possible. The problem that arises in LMMSE channel estimation is to compute correlation matrix. Different techniques are used to calculate the correlation matrix (assuming power delay profile (PDP) is known). In [3], discrete Fourier transform (DFT) based approach is used to calculate the correlation matrix. However, in some cases PDP is unknown. In this case, some distribution for PDP is assumed and then the correlation matrix is calculated. Different solution are proposed for the same in [29], [30], [21]. Estimation performance of estimator can further be improved by using correlation in both the time-domain and the frequency-domain, and this type of estimator are called as 2D-LMMSE estimator [17]. 2D LMMSE estimator also has high computational complexity. Complexity of 2D LMMSE estimator can be reduced by computing correlation matrix for the time-domain and the frequency–domain separately [31] [21]. Channel estimation with null subcarriers is proposed in [32], channel estimation is first perform in frequency domain and then transformed back in time domain to reduce

interpolation error. In [33] various interpolation techniques are used to estimate the channel utilizing the knowledge of pilot symbols.

In all the above discussed channel estimation techniques, some prior information of channel is required for better performance. In practice, this information is generally not available, and it also varies with time. Instead of using prior knowledge of channel, there are many other ways to estimate the channel. One of the ways is to use adaptive filters. Adaptive filters can follow the channel statistics and adjust its coefficients accordingly. In [34], [35], two dimensional adaptive channel estimation algorithm, based on 2D- normalized least mean square (NLMS) adaptive filter is used. 2D-NLMS adaptive filter differs from 1D-NLMS adaptive filters only in one aspect that it takes advantage of both the time-domain and the frequency-domain correlation. These filters are more suitable for doubly selective (time selective and frequency selective) fading channels. 2D-NLMS channel estimator has slow convergence, which means it needs large number of OFDM symbols to converge. In [36], [37], two dimensional recursive least square adaptive filter and Kalman filter are used to track the channel statistics. These two estimator have fast convergence, but also have high computational complexity. Channel estimation based on set-membership NLMS algorithm is proposed in [38]. In [39], a complex block LMS algorithm is proposed which is based on gradient algorithm. Adaptive filters can track the channel statistics more accurately, but they still require few OFDM symbols to converge. Thus, they are only suitable for those channels in which Doppler spread is very much less.

1.3 Objective of thesis

The main objective of this thesis is to investigate OFDM as a modulation technique for wireless radio propagation with major channel factors (delay spread, fading etc.) affecting the digital communication system and to show OFDM system performance in the above conditions. Channel estimation techniques for OFDM system are investigated by proposing two new approaches.

1.4 Outline of thesis

The rest of thesis is organized as follows:

- **Chapter 2:** Multicarrier modulation scheme is described and OFDM system and efficient implementation of the OFDM systems through DFT/IDFT is presented. Multipath channel and various terminologies of multipath channel is also explained.

- **Chapter 3:** Here various channel estimation techniques such as least squared (LS), minimum mean squared error (MMSE) estimation techniques are explained, we then develop channel estimation method based on regularized least squared estimation. Simulations are performed to implement the proposed method and the performance of proposed method is compared with the existing methods.

- **Chapter 4:** Adaptive channel estimation methods based on least mean squared (LMS) adaptive filters, normalized least mean squared (NLMS) adaptive filters are explained. An Adaptive channel estimator based on modified NLMS adaptive filter with variable step size is developed. Simulations are performed to implement the proposed method and the performance of proposed method is compared with earlier suggested methods.

- Finally the conclusion and scope for future work are presented in **chapter 5**

Chapter 2

Principles of OFDM

2.1 Multi-carrier Modulation System

OFDM is a concept taken from multi-carrier modulation and demodulation technique [40]. A simple multi-carrier modulation technique is based on frequency division multiplexing (FDM), in which available wide band is divided into large number of narrow non overlapping sub-bands. Data are transmitted in parallel with the help of these narrow sub-bands. Ideally these narrow sub-bands are narrow enough so that sub-band channels can be considered as slow and flat fading channel and ISI can be minimize. Fundamental block diagram of multi-carrier system is shown in figure 2.1.

Input data stream is divided into small sub-data streams and these sub-data streams are mapped into desired waveforms followed by filter banks, which basically limits the bandwidth of the incoming signal. After passing through filter banks, these waveforms modulate the sub-carriers having different center frequencies, which are then multiplexed and transmitted. Figure 2.2 shows the spectrum allocation of sub-carriers in frequency division multiplexing (FDM) system, where $f_0, f_1, \ldots, f_{N-1}$ are the center frequencies of the sub-carriers. Consider the input sequence to be $a[k]$ for $k = 0, 1, \ldots, N-1$ and frequency spacing between each sub-carrier is Δf and symbol interval is T_s, then transmitted symbol in multi-carrier modulation scheme is given

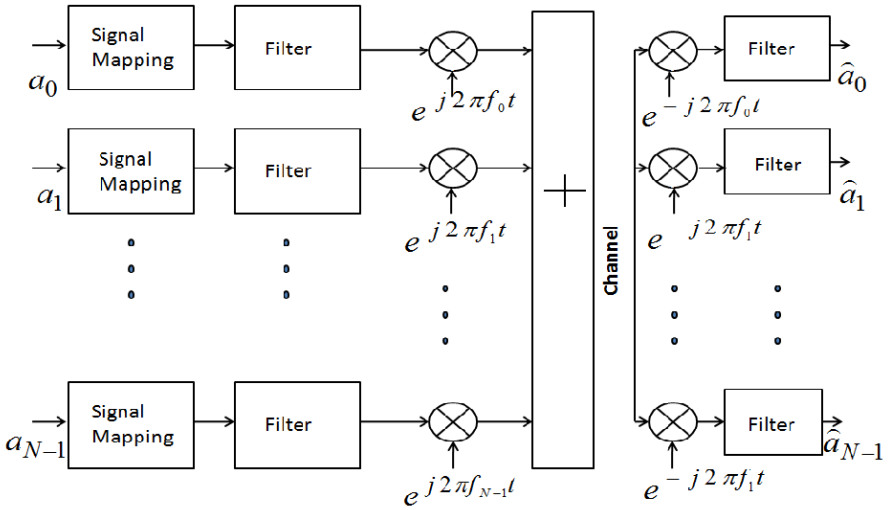

Figure 2.1: Block diagram of typical multi-carrier system

by

$$x(t) = \sum_{N-1}^{k=0} a[k] e^{j2\pi k \Delta f t}, 0 \leq t \leq T_s. \tag{2.1}$$

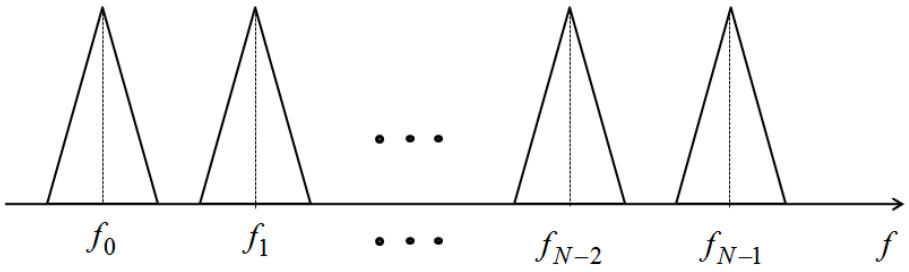

Figure 2.2: FDM sub-band spectrum distribution

Above modulation scheme has some disadvantages:

a. Since sub-carriers are not allowed to overlap with each other, the wide spacing between sub-bands means a lower spectral efficiency.

b. Use of filter banks at transmitter and receiver makes system more complex.

2.2 OFDM principle

In multi-carrier transmission system, if we allow the sub-bands to overlap then spectral efficiency can be increase. This is the fundamental concept of orthogonal frequency division multiplexing (OFDM). So OFDM system can be considered a special case of multi-carrier system, in which sub-bands are allowed to overlap. Now the transmitted signal in multi-carrier system given by

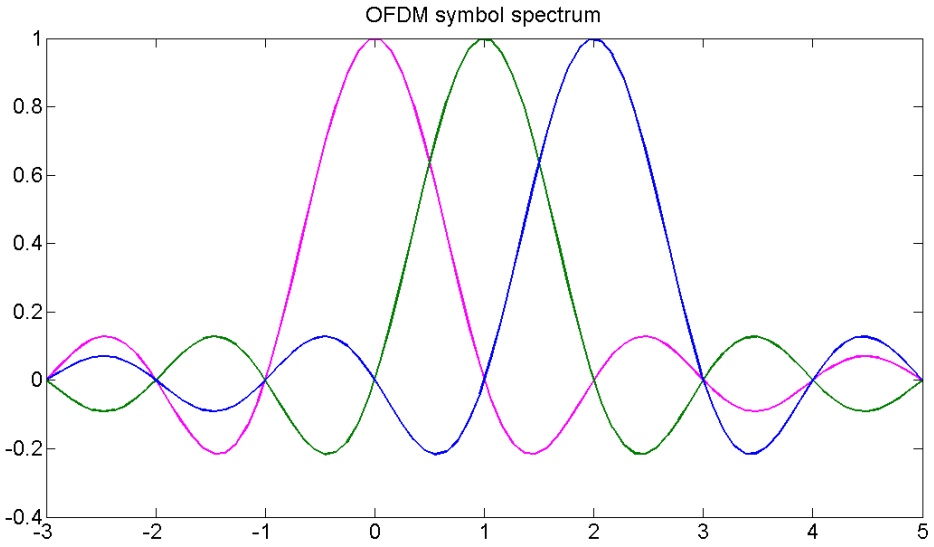

Figure 2.3: Overlapping spectrum of OFDM

$$x(t) = \sum_{N-1}^{k=0} a[k] e^{j2\pi k \Delta f t}, 0 \leq t \leq T_s. \tag{2.2}$$

Spectrum can be allowed to overlap if any two sub-carriers are orthogonal to each other. Let us first introduce the concept of orthogonality. Two signals are said to be orthogonal, if their inner product is zero. If ψ_l and ψ_m are the two signal, then, they are orthogonal if below equation is satisfied

$$\int_{(\alpha)T_u}^{(\alpha+1)T_u} \psi_l(t)\, \psi_m^*(t)\, dt = 0. \tag{2.3}$$

where $\alpha T_u \leq t \leq (\alpha+1)T_u$ and $l \neq m$

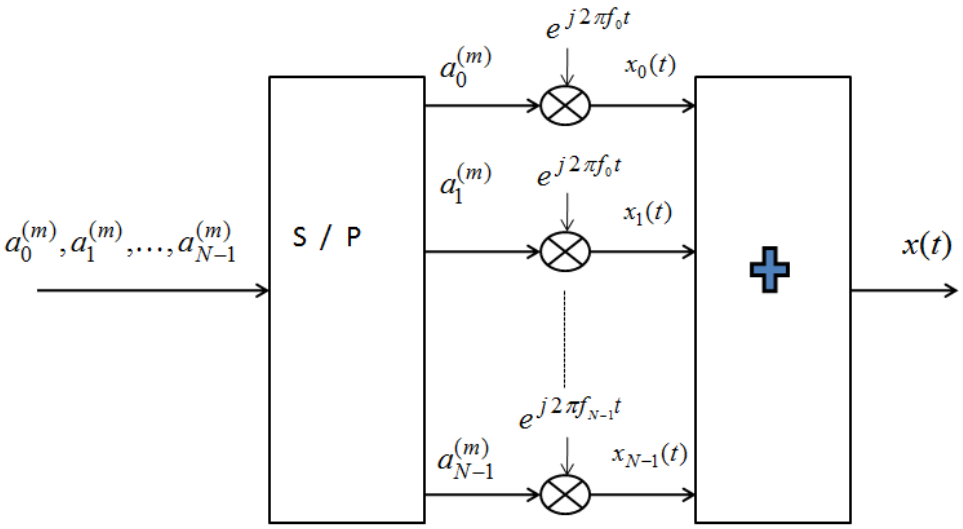

Figure 2.4: OFDM modulation

Consider the sub-carriers used in multi-carrier system, $e^{j2\pi k\Delta f t}$ for $k = 0, 1, \ldots, N-1$. Any two complex exponential signals are orthogonal if their center frequencies are integer multiple of fundamental frequency.

2.3 OFDM Implementation Using IFFT and FFT processor

Although bank of correlators or modulators can be use to illustrate the basic principle of OFDM modulation and demodulation, but using such a large number of correlators makes the OFDM system more complex. OFDM system can be made simple due to its beautiful structure and selection of sub-carrier spacing Δf, hence FFT implementation is possible. Consider an OFDM signal,

$$x(t) = \sum_{k}^{N-1} a[k] e^{j2\pi k\Delta f t} \quad , \qquad 0 \leq t \leq T_s \tag{2.4}$$

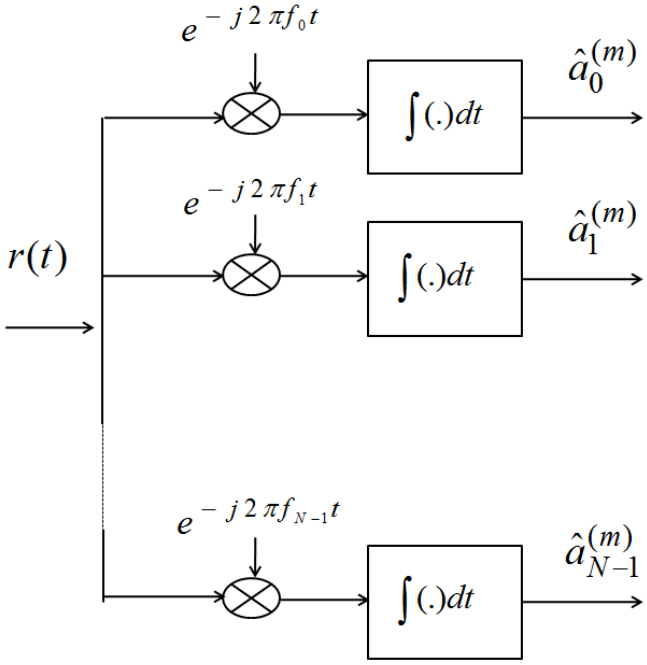

Figure 2.5: OFDM demodulation

If the signal is sampled at a rate of $\frac{T_s}{N}$, then above equation can be rewritten as

$$x[n] = x[\frac{nT_s}{N}] = \sum_{k}^{N-1} a\,[k]\, e^{\frac{j2\pi k \Delta f T_s}{N}} \tag{2.5}$$

Now if following equation,

$$\Delta f T_s = 1 \tag{2.6}$$

is satisfied then sub-carriers are orthogonal to each other and equation 2.5 can be rewritten as

$$\begin{aligned} x\,[n] &= \sum_{k=0}^{N-1} a\,[k]\, e^{j2\pi nk/N} \\ &= N.\text{IDFT}\ a[k] \end{aligned} \tag{2.7}$$

The above equation is the IDFT expression of the input symbols $a[k]$. At the receiver DFT of received signal should come out to be the transmitted signal $x[n]$

Suppose $\hat{a}\,[k]$ is the DFT of transmitted signal, i.e.

$$\hat{a}[k] = DFT\{x[n]\}$$

$$= \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} e^{j2\pi n(m-k)/N}$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} a[m] \sum_{n=0}^{N-1} e^{j2\pi n(m-k)/N}$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} a[m]\delta[m-k]$$

Now if $m = k$, then $\hat{a}[k] = a[k]$. Suppose $m \neq k$, which means sub-carrier at time of modulation and sub-carrier at the time of demodulation are different hence

$$\hat{a}[k] = 0$$

which must be true since carriers are orthogonal. Hence, from the above discussion it is clear that modulator and demodulator for OFDM can be implement using IFFT and FFT.

2.4 Cyclic Prefix

Time dispersive channel causes ISI. In OFDM system orthogonality of sub-carriers is also necessary. Due to time dispersive channel orthogonality of sub-carriers also get lost, which is called inter carrier interference (ICI). To overcome the problem of ICI cyclic prefix (CP) is inserted into OFDM symbol before transmission. Cyclic prefix is the copy of some previous part of the OFDM symbol. Cyclic prefix are removed at the receiver before demodulation.

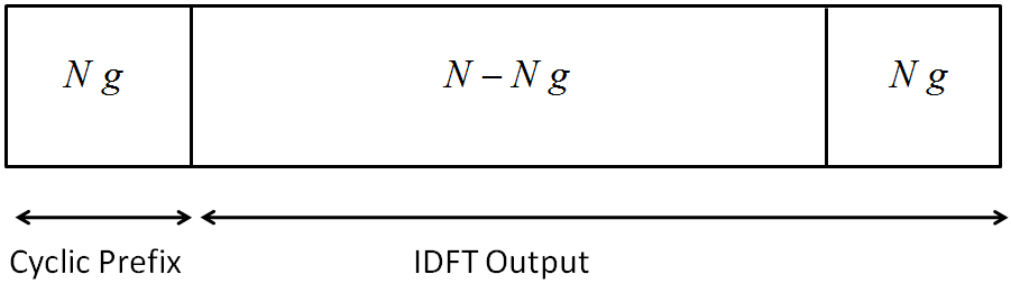

Figure 2.6: cyclic prefix in OFDM system

Insertion of CP has many advantages, such as: 1) it converts the linear convolution with channel impulse response into circular convolution and 2) if length of CP is longer than the length of channel impulse response, then ISI can be removed, Since CP is acting as time guard between two consecutive OFDM symbols. As circular convolution in time domain is equivalent to multiplication in frequency domain, which means sub-carrier remains orthogonal and ICI is also removed. The only disadvantage of CP is that some bandwidth is waste as CP does does not contain any information.

2.5 Frequency domain model of OFDM transmission

In OFDM system, time dispersive channel and OFDM demodulation can be seen as frequency domain channel as shown in figure 3.2. Where frequency domain channel attenuations are given by $H_0, H_1, \ldots H_{N-1}$. Demodulator output b_k is the transmitted signal a_k scaled by channel impulse response and added noise n_k. Hence, exact recovery of transmitted signal requires only one tap equalizer per sub-carrier.

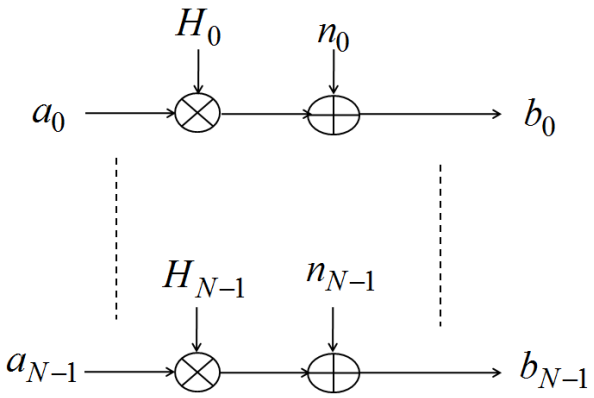

Figure 2.7: Frequency domain model of OFDM system

2.6 Advantages and Disadvantages of OFDM

2.6.1 Advantages

a. For a given channel Nyquist rate can be achieved easily without use of sharp cut off filters.

b. OFDM is an efficient way to deal with multipath; also implementation complexity of the equalizer is very low as compared to single carrier system.

c. In slow time varying channels, performance of OFDM system can be enhanced by adapting the data rate according to signal to noise ration (SNR) for that sub-carrier.

d. OFDM is more robust against narrow band interference since such interference affects only small number of sub-carriers.

2.6.2 Disadvantages

OFDM has some disadvantages also, which are.

a. OFDM is more sensitive to frequency offset and phase noise.

b. OFDM has relatively large peak to average power ratio (PAPR).

2.7 Channel model

Channel is the medium between transmitter and receiver; it can be a wire or free space. Free space, which is more commonly called as wireless channel uses electromagnetic waves to carry the signal. Most common channel model used is Gaussian model, which is commonly known as Additive white Gaussian noise (AWGN) channel. When signal is transmitted through this channel it is corrupted by Gaussian noise. This channel model assumes thermal noise is the only disturber to the front end of the receiver. Thermal noise has flat power spectral density. AWGN channel model can be consider as good model for wired channel, but when it comes to wireless environment there are many parameters which are more significant than thermal noise. In wireless environment path through which signal can travel is not one, but there more than one, and also these paths are not fixed, hence multi path fading channel model is better model for wireless channels. When a signal travels through radio channel it suffers from reflection, scattering, diffraction, attenuation, and delay. At the receiver same signal traveled from different paths may add up in constructively or destructively, depending on the phase provided by each path. Let us first define some important terminology of multipath channel.

2.7.1 Impulse response

The complex impulse response $h(t)$ of a time invariant multipath channel having N paths can be represented as

$$h(t) = \sum_{k=0}^{N-1} a_k e^{j\theta_k} \delta(t - \tau_k), \tag{2.8}$$

where k is the path index, a_k is the attenuation provided by k^{th}path, θ_k is the phase of k^{th} path, and τ_k is the delay of k^{th} path. Above model does not consider the time varying nature of the channel, this model is suitable for OFDM system.

2.7.2 Power Delay profile

Power delay profile (PDP) is defined as absolute squared value of the channel impulse response

$$P(t) = \sum_{k=0}^{N-1} a_k^2 \delta(t - \tau_k) \tag{2.9}$$

where $P(t)$ is power delay profile, τ is time delay, k is tap delay index, a_k is attenuation of the channel, and N is total number of path. Power delay profile represents the relative received power as a function of the excess delay with respect to the first received path.

2.7.3 RMS delay spread

Delay spread of the channel can be defined as time spread between arrival of the first and the last received signal [41]. It represents the spread in time over which multipath signal arrive and measure of time dispersion. Mathematically delay spread is defined by the term root mean squared (RMS) delay spread. RMS delay spread is the measurement of delay spread of multipath channel. To define RMS delay spread Consider a multipath fading channel whose impulse response can be modelled as

$$h(t) = \sum_{k=0}^{N-1} a_k e^{j\theta_k} \delta(t - \tau_k), \tag{2.10}$$

then RMS delay spread is define as

$$\tau_{rms} = \sqrt{\frac{\int (t - \bar{\tau})^2 P(t) dt}{\int P(t)}} \tag{2.11}$$

where

$$\bar{\tau} = \frac{\int tP(t)dt}{\int P(t)dt}.$$

τ_{rms} can be used as an approximation of maximum data rate which can be reliably supported by the channel without use of equalization.

$P(t)$ is the power delay profile (PDP) represented as,

$$P(t) = \sum_{k=0}^{N-1} a_k^2 \delta(t - \tau_k)$$

2.7.4 Coherence bandwidth

The coherence bandwidth of the channel denoted by B_{coh} gives a measure of the statistical average bandwidth for which channel characteristics are correlated.
Let us start by defining the autocorrelation of a channel frequency response from its statistical expectation.

$$R(\Delta f) = E[H(f)H^*(f + \Delta f)] = \frac{1}{a + j2\pi\tau_{rms}\Delta f} \tag{2.12}$$

and

$$R(\Delta f) = E\{|H(f)||H^*(f + \Delta f)|\} = \frac{1}{1 + (2\pi\tau_{rms\Delta f})^2} \tag{2.13}$$

where $H(f)$ is the frequency response of the time-invariant multipath channel. Now B_c can be defined as the value of Δf for which $R(\Delta f)$ get decreased by 3 dB

$$\frac{R(\Delta f)}{R(0)} = \frac{1}{2} \quad for \quad which \quad \Delta f = B_c \tag{2.14}$$

2.7.5 Coherence time

The coherence time T_c is the time for which the correlation of the channel impulse response decreased by 3 dB. The time correlation is defined as

$$R(\Delta t) = \int h(t)h^*(t + \Delta t)dt \tag{2.15}$$

Coherence time is a statistical measure of time duration over which the channel impulse response is invariant, hence coherence time is the time duration over which two received signal are highly correlated in terms of amplitude. As the time between

two channel impulse responses increases, the correlation generally decreases.

2.7.6 Doppler Spectrum

Doppler spectrum is the Fourier transform of time correlation function,

$$P_h(f) = \int_{-\infty}^{\infty} R(\Delta t) e^{-j2\pi ft} d\Delta t. \tag{2.16}$$

From above equation Doppler spread can be define, Doppler spread B_D is defined as the range of frequencies over which received Doppler spectrum is non-zero. Coherence time and Doppler spread are related to each other by below equation

$$T_{coh} = \frac{1}{B_D} \tag{2.17}$$

2.7.7 Multipath Fading

Different types of channels can be defined according to the terminologies we have defined so far. Let us first characterize the channel in frequency domain. Two types of channels can be defined in frequency domain and two types in time domain

Flat fading channel

It basically means what its name says. This type of channel has linear phase which means constant group delay. Group delay is a measure of channel phase distortion, and can be defined as the derivative of the channel phase characteristics with respect to frequency. When bandwidth B_s of the signal is less than the coherence bandwidth B_c of the channel, then channel is said to be flat fading channel

$$B_s << B_c \tag{2.18}$$

This type of channel can be particularly seen in OFDM system since each subbands are so narrow that they can be considered flat.

Frequency selective channel

It is opposite to frequency flat fading channel, it is usually referred to as wideband channel because the bandwidth of the signal is wider than the bandwidth of the available channel.

$$B_s > B_c \tag{2.19}$$

In frequency selective fading the spectral components of signal undergoes attenuation by different factor.

Slow Fading channel

If the channel impulse response changes slowly as compared to symbol period of signal T_s, then channel is said to be slow fading channel.

$$T_s << Tc \tag{2.20}$$

In slow fading, the channel impulse response can assume to be constant over symbol period

Fast fading channel

If the channel impulse response changes fast as compared to symbol period of signal T_s then channel is said to be fast fading channel.

$$T_s > T_c \tag{2.21}$$

In fast fading channel, the channel impulse response cannot be consider constant for one symbol period

2.8 Steepest descent method

Steepest descent algorithm is a recursive algorithm which tracks the time variation of the signal statistics and gives the solution without solving Wiener-Hopf solution for each time the statistics change[42]. Consider a cost function $J(\omega)$, which is function of some unknown vector and maps all the elements of ω into real number. We are interested in that value of ω for which $J(\omega)$ is minimum (called as optimum solution). If ω_0 is our optimum solution then

$$J(\omega_0) \leq J(\omega) \qquad \forall\omega$$

Optimum solution for above cost function $J(\omega)$ can be obtained in iterative way, starting with an initial guess of $w(0)$ try to update the unknown vector ω in iterative way so that cost function reduces at after each iteration

$$J(w(n+1)) \leq J(w(n))$$

Where $w(n)$ is the old guess of unknown vector ω and $w(n+1)$ is the new guess. In steepest descent algorithm unknown weight vector is updated in the direction opposite to the direction of gradient vector of cost function $J(w)$. If $\nabla J(\omega)$ is the gradient of $J(w)$ then

$$\nabla J(\omega) = \frac{\partial}{\partial\omega} J(w)$$

So that steepest descent algorithm can be given by

$$w(n+1) = w(n) - \mu\nabla J(w(n)) \tag{2.22}$$

where μ is called as step-size, it controls the convergence and stability of the algorithm toward optimum solution. It has been shown that μ has some range given by [42]

$$0 < \mu < \frac{2}{\lambda_{max}} \tag{2.23}$$

where λ_{max} is the maximum eigen value of the input vector.

Chapter 3

Channel estimation

Channel estimation is an important process in receiver design of OFDM systems. In order to compensate the channel effects on the received signal, channel estimation is required to provide information for further processing of the received signal. In this chapter, only pilot based channel estimation is explained. Pilot symbols are arranged

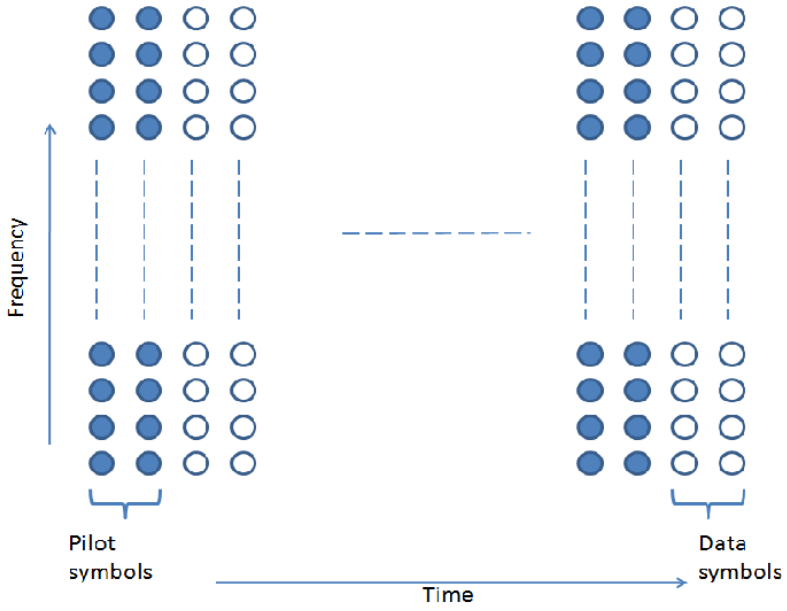

Figure 3.1: Block type pilot arrangement

in block-type manner as shown in figure 3.1.

In the block-type arrangement, one specific symbol full of pilot subcarrier is transmitted periodically. A receiver can obtain estimate for channel gains at all subcarriers from this pilot symbol and then apply estimated channel gains to equalize received data symbols. Since channel is updated periodically, block-type arrangement is suitable for slow fading channel. For fast fading channels comb-type arrangement is used [43], [26].

3.1 Pilot Based Channel Estimation

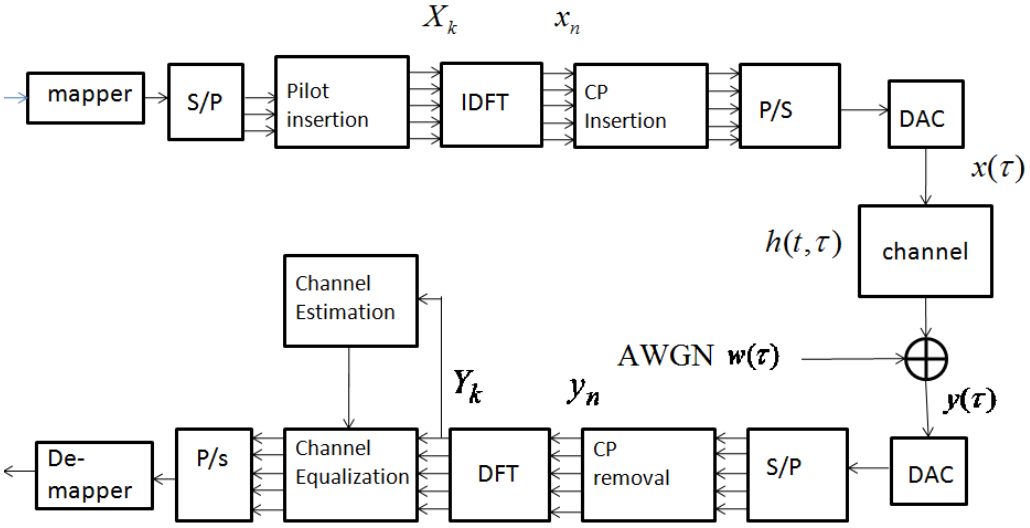

Figure 3.2: Block diagram of Pilot aided OFDM system

Figure 3.2 shows the model for pilot aided baseband OFDM system [26]. In the transmitter, incoming data stream is grouped in blocks of N data symbol. These groups are called as OFDM symbols and can be represented by a vector $\mathbf{x}$ given by $\mathbf{x} = [X_0, X_1, \ldots, X_{N-2}, X_{N-1}]^T$, where, X_k represents data symbol corresponding to k^{th} subcarrier. Next, an IDFT is performed on each data symbol block, X_k, and a cyclic prefix of length N_g is added.

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N} \tag{3.1}$$

where $n = -N_g, \ldots, 0, \ldots, N-1$ and N represents total number of sub-carriers. The length of cyclic prefix, N_g, should be greater than length of channel impulse response. Consider equation 2.8, which represent the impulse response of the multipath channel,

$$h(t) = \sum_{k=0}^{N-1} a_k e^{j\theta_k} \delta(t - \tau_k).$$

Above equation can also be written as [26]

$$h(t, \tau) = \sum_{r}^{i=0} h_r(t) \cdot \delta(\tau - \tau_r), \tag{3.2}$$

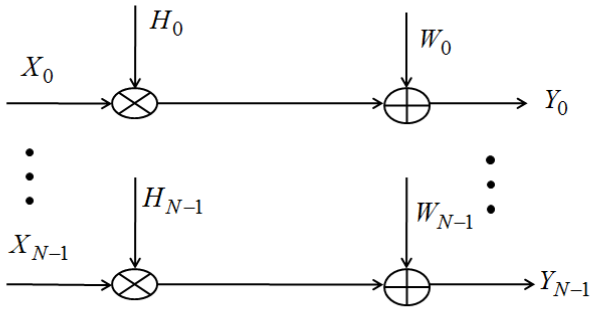

Figure 3.3: Parallel Gaussian channels

where h_r(t) is the path gain, and is wide sense stationary (WSS) narrow-band complex Gaussian process and are mutually independent, τ is delay spread index, and τ_r is the delay of r^{th} path. If length of cyclic prefix is longer than length of the channel impulse response, then multipath OFDM channel can be considered a set of parallel Gaussian channel (a complex gain followed by additive white Gaussian noise)

as shown in figure 3.3, that is free of ISI and ICI [44]. Assume that the channel is almost stationary within one symbol period, i.e. $h_r(t) = h_r$; then, the frequency-domain channel response at k^{th} subcarrier can be represented as H_k. Received signal corrupted by additive white Gaussian noise (AWGN) and multipath fading channel after passing through DFT block is given by

$$Y_k = X_k H_k + W_k \tag{3.3}$$

where Y_k is DFT of received signal given by

$$Y_k = \sum_{n=0}^{N-1} y_n \cdot e^{-j2\pi nk/N} \tag{3.4}$$

and

$$H_k = \sum_{i=0}^{r} h_r e^{j2\pi\tau_r k/T_s N}, \tag{3.5}$$

where T_s is sample interval, and W_k is defined as the DFT of added noise.

Define a DFT matrix, $\mathbf{F}$, as

$$\mathbf{F} = \begin{bmatrix} W_N^{00} & \dots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \dots & W_N^{(N-1)(N-1)} \end{bmatrix} \tag{3.6}$$

where

$$W_N^{nk} = \frac{1}{N} e^{-j2\pi(n/N)k} \tag{3.7}$$

Using linear algebra concepts equation 3.5 can be written as

$$\mathbf{y} = \mathbf{XFh} + \mathbf{w} \tag{3.8}$$

where $\mathbf{y}$ is a vector containing received OFDM symbol given by

$$\mathbf{y} = [Y_0, Y_1, \ldots, Y_{N-2}, Y_{N-1}]^T,$$

$\mathbf{X}$ is diagonal matrix containing data symbol blocks given by

$$X = diag[X_0, X_1, \ldots, X_{N-2}, X_{N-1}],$$

$\mathbf{g}$ is discrete time domain channel impulse response given by

$$\mathbf{g} = [g_0, g_1, \ldots, g_{N-2}, g_{N-1}]^T,$$

$\mathbf{h}$ is frequency-domain channel response given by,

$$\mathbf{h} = [H_0, H_1, \ldots, H_{N-2}, H_{N-1}]^T,$$

and $\mathbf{w}$ is the DFT of added noise,

$$\mathbf{w} = [W_0, W_1, \ldots, W_{N-2}, W_{N-1}]^T.$$

Suppose that the channel responses at all data subcarriers are estimated using pilot subcarriers and are denoted as $\hat{H}_k$, $k = 0, 1, \ldots, N-1$; then data cab be equalized by

$$\hat{X}_k = \frac{Y_k}{\hat{H}_k}, \quad k = 0, 1, \ldots, N-1. \tag{3.9}$$

.

3.1.1 Least-Squares Estimator

Least squares (LS) channel estimator is the simplest channel estimator, which assumes that the time domain channel impulse response is deterministic and tries to find $\hat{\mathbf{g}}_{LS}$

which minimize $(\mathbf{y} - \mathbf{XFg})^H(\mathbf{y} - \mathbf{XFg})$. The LS solution is then given by [44]

$$\hat{\mathbf{g}}_{LS} = (\mathbf{F}^H\mathbf{X}^H\mathbf{XF})^{-1}\mathbf{F}^H\mathbf{X}^H\mathbf{y}. \tag{3.10}$$

The frequency-domain channel impulse response can be found by taking DFT of $\hat{\mathbf{g}}_{Ls}$, which is given by

$$\hat{\mathbf{h}}_{LS} = \mathbf{F}(\mathbf{F}^H\mathbf{X}^H\mathbf{XF})^{-1}\mathbf{F}^H\mathbf{X}^H\mathbf{y}. \tag{3.11}$$

Since in block type pilot symbols, all the subcarriers are used to carry pilot symbols, matrices $\mathbf{F}$ and $\mathbf{X}$ are invertible square matrices; hence equation 3.11 can be simplify as

$$\hat{\mathbf{h}}_{LS} = \mathbf{X}^{-1}\mathbf{y}. \tag{3.12}$$

3.1.2 Minimum Mean Squared Error Estimator

In this method, equivalent time domain channel impulse response $\mathbf{h}$ is assumed to be a random vector. Assume that time-domain channel impulse response, $\mathbf{h}$, is a random vector with Gaussian distribution and uncorrelated with noise $\mathbf{W}$. Then, minimum mean squared error (MMSE) estimator that minimizes $E\{(\hat{\mathbf{h}} - \mathbf{h})^H(\hat{\mathbf{h}} - \mathbf{h})\}$ is given by [44]

$$\hat{\mathbf{g}}_{MMSE} = \mathbf{R}_{gy}\mathbf{R}_{yy}^{-1}\mathbf{y} \tag{3.13}$$

where

$$\mathbf{R}_{gy} = E\{\mathbf{g}\mathbf{y}^{\mathbf{H}}\} = \mathbf{R}_{gg}\mathbf{F}^H\mathbf{X}^H \tag{3.14}$$

is the cross correlation between $\mathbf{g}$ and $\mathbf{y}$; and,

$$\mathbf{R}_{\mathbf{yy}} = E\{\mathbf{y}\mathbf{y}^H\} = \mathbf{XFR}_{\mathbf{gg}}\mathbf{F}^{\mathbf{H}}\mathbf{X}^{\mathbf{H}} + \sigma_w^2\mathbf{I}_N \tag{3.15}$$

is the auto-covariance matrix of $\mathbf{y}$; $\mathbf{R_{gg}}$ is the auto-covariance matrix of $\mathbf{g}$, which is assumed to be known in advance. The MMSE frequency-domain channel response $\hat{\mathbf{h}}_{MMSE}$ is given by

$$\hat{\mathbf{h}}_{MMSE} = \mathbf{F}\hat{\mathbf{g}}_{MMSE}. \tag{3.16}$$

Since $\hat{\mathbf{g}}_{MMSE}$ is linear function of $\mathbf{y}$, this estimator is also a linear minimum mean squared error (LMMSE) estimator. The MMSE channel estimator has high computational complexity, but good performance in terms of MSE. To reduce the complexity of MMSE channel estimator, low rank approximation is proposed in [28]. In this method, rank of $\mathbf{R}_{hh}$ is reduced using *eigen decomposition*.

3.2 Proposed Method

Least squared estimation method used in channel estimator can be more powerful, if in addition to error some more constraints are also minimized. Popular approach is regularized least squared method, which tries to minimize error along with some other constraints.

3.2.1 OFDM Channel estimator based on Regularized Least Squared Estimation method

Regularized least square is more powerful method as it considers some constraints, there are various specifications of regularized least square [45]. In proposed method, some prior information of channel is used as a constraint in regularized least square method, prior information is assumed to be the distribution of CSI. Assume that the probability mass function (PMF) of CSI is $P_G(g)$, then the histogram of this PMF can be represented by vector $\mathbf{g}_{hist}$ of order $p \times 1$. Now, recall that received OFDM symbol in frequency domain is given by,

$$\mathbf{y} = \mathbf{XFg} + \mathbf{n}$$

Now, apply regularized least squared estimation technique, considering g_{hist} as a constraint, we have

$$J(\hat{\mathbf{g}}) = \|\mathbf{y} - \mathbf{X}\mathbf{F}\hat{\mathbf{g}}\|^2 + \lambda\|\hat{\mathbf{g}}_{hist} - \mathbf{g}_{hist}\|^2 \tag{3.17}$$

where, $J(\hat{\mathbf{g}})$ is the cost function which need to be minimize, $\hat{\mathbf{g}}$ is estimated channel impulse response, $\hat{\mathbf{g}}_{hist}$ is the histogram of estimated channel impulse response, λ is the weight factor, and $\mathbf{g}_{hist}$ is histogram of PMF of channel impulse response, which is assumed to be known. Since the cost function $J(\hat{\mathbf{g}})$ is a convex function, it can be minimized using steepest descent method. The value of $\hat{\mathbf{g}}$ for which cost function is minimum is considered as estimated channel impulse response. Gradient of cost function can be calculated by differentiating the cost function $J(\hat{\mathbf{g}})$, i.e.

$$\nabla J\left(\hat{\mathbf{g}}\right) = \left(\mathbf{Q} + \mathbf{Q}^H\right)\hat{\mathbf{g}} - \hat{\mathbf{g}}^H\mathbf{F}^H\mathbf{y} \tag{3.18}$$

where $\mathbf{Q}$ is a matrix of order $N \times N$ given by

$$Q = \mathbf{F}^H\mathbf{F}\mathbf{X}^H\mathbf{X} \tag{3.19}$$

After each iteration, $\hat{\mathbf{g}}$ is updated by

$$\hat{\mathbf{g}}\left(k+1\right) = \hat{\mathbf{g}}\left(k\right) - \mu\nabla J\left(\hat{\mathbf{g}}\left(k\right)\right)$$

or

$$\hat{\mathbf{g}}\left(k+1\right) = \hat{\mathbf{g}}\left(k\right) - \mu\left[\left(\mathbf{Q} + \mathbf{Q}^H\right)\hat{\mathbf{g}}\left(k\right) - \mathbf{g}^H\left(k\right)\mathbf{F}^H\mathbf{y}\right] \tag{3.20}$$

where, μ is the stepsize. It can be a constant or variable, variable stepsize lead to fast convergence of cost function towards optimum solution. The value of $\hat{\mathbf{g}}$ for which cost function $J(\hat{\mathbf{g}})$ is minimum, estimated channel impulse response $\hat{\mathbf{g}}_{est}$. Cor-

responding frequency-domain channel response can be by taking DFT of $\hat{\mathbf{g}}_{est}$ i.e.

$$\hat{\mathbf{h}}_{est} = \mathbf{F}\hat{\mathbf{g}}_{est}. \tag{3.21}$$

This $\hat{\mathbf{h}}_{est}$ is then can be used by equalizer for further processing.

3.3 Simulation results and comparisons

Pilots are arranged in block type scheme, First the data streams (pilot symbols) are mapped through 64 QAM modulation techniques, and output of QAM modulation is named as QAM symbols. These QAM symbols are grouped in N blocks and then IDFT operation is performed. The cyclic prefix are then added to IDFT output, which are then serially converted and transmitted. Additive noise is assumed to be Complex normal Gaussian (iid) noise. Prior distribution of channel impulse response is assumed to complex normal Gaussian distributed and length of channel impulse response is assumed to be 16. In steepest descent method, variable step size is used, for first half of iteration value of stepsize is 0.25, while for second half of iteration value of stepsize is 0.085. Mean square error is calculated using

$$MSE = E[|\mathbf{h} - \hat{\mathbf{h}}_{est}|^2] \tag{3.22}$$

where $\hat{\mathbf{h}}_{est}$ is the estimated channel vector and $\mathbf{h}$ is the true channel vector. The symbol error rate is calculated for data symbol according to

$$SER = \frac{Total\ error\ symbols}{Total\ data\ symbols\ sent} \tag{3.23}$$

Parameter	**Value**
Sampling rate $F_0 = 1/T$	20 MHz
FFT size N	64
Carrier central frequency f_c	5.2 GHz
Useful symbol part duration T_u	3.2 μs
Cyclic prefix length N_{CP}	16
Cyclic prefix duration T_{CP}	0.8 μs
Symbol interval $T_s = T_u + T_{CP}$	4μs
Sub-carrier spacing $F = \frac{1}{T_u}$	0.3125 μs
Data symbol constellation	64 QAM

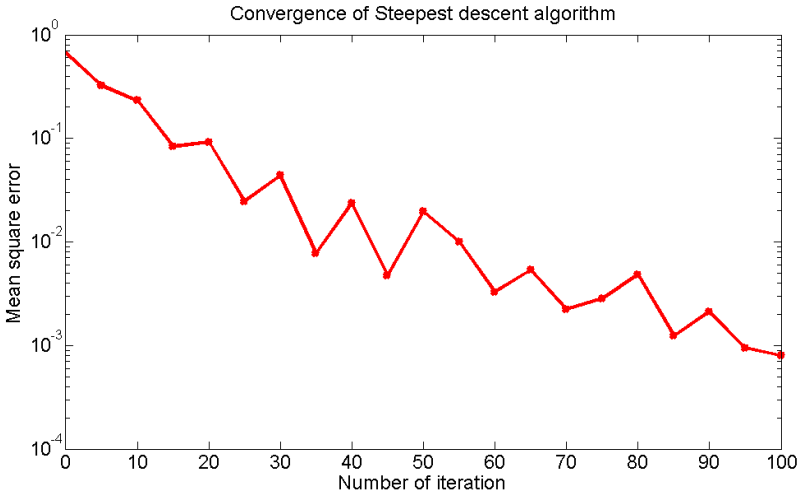

Figure 3.4: Convergence of steepest descent algorithm

MSE and BER of proposed method is compared estimation technique proposed in [44], Figure 3.4 shows convergence of MSE per iteration. Figure 3.5 and figure 3.6 shows the MSE and SER comparison of proposed channel estimator with LS channel estimator [44] From simulation results it is clear that Regularized least square method works better as compared LS method for given SNR. So above estimator are suitable for the high SNR channels where complex estimators are not viable.

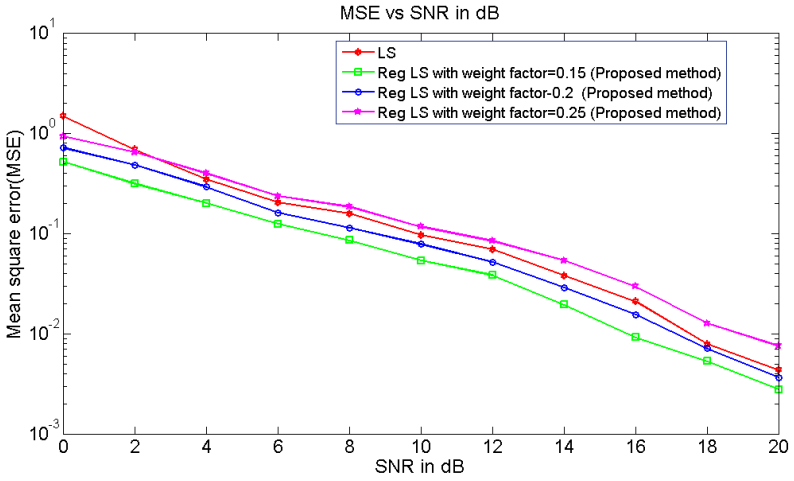

Figure 3.5: Plot between bit Mean square error (MSE) and SNR in dB

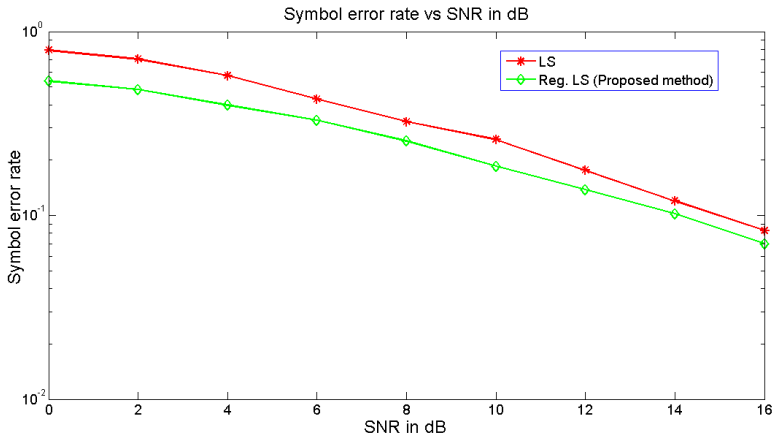

Figure 3.6: Plot of symbol error rate (SER) vs SNR in dB

Chapter 4

Adaptive channel estimation

Adaptive signal processing is a fast emerging field widely used in system identification. As input to the system varies with time, digital filters with fixed coefficients are not suitable since they require well defined input signal. Adaptive signal processing or often called as adaptive filters are the time variant filters which changes its characteristics according to the input signal. Since channel estimation in OFDM system is equivalent to system identification, adaptive filters are suitable as input signal and the channel is much more unpredictable.

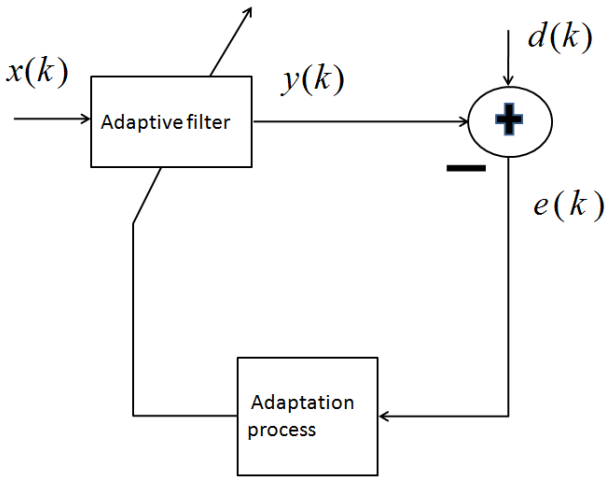

Figure 4.1: General adaptive signal processing configuration

Figure 4.1 shows a typical adaptive system in which given reference signal $d(k)$,

adaptive filter adjust its coefficients for any input $x(k)$, such that error signal $e(k)$ is minimum. Channel estimation in OFDM systems can be done using adaptive filters. Adaptive filter adjust its coefficients according to each incoming OFDM symbol, filter coefficients then can be use as channel impulse response for further equalization process. Adaptive signal processing or adaptive filters are based on adaptive algorithms

4.1 Adaptive Filters

There are two popular adaptive filters namely, least-mean squared (LMS) adaptive filter and recursive least-squared (RLS) adaptive filter. LMS adaptive filter is described in next section. RLS adaptive filters can be found in [42].

4.1.1 LMS adaptive filter

LMS filters are based on linear adaptive filtering algorithm which consists of two processes:

a. A filtering process, which involves computing the output of an adaptive filter in respond to input signal and then generating the error signal with reference to desire signal.

b. An adaptive process, which involves adjustment of linear filter coefficients to minimize the error signal

LMS adaptive filters can be applied to both real valued signal and complex valued signal, Since in any digital communication system processing is done on baseband signal so complex form of LMS algorithm is explained. Recall Wiener Hopf solution [42]

$$\mathbf{w}_0 = \mathbf{R}^{-1}\mathbf{p}. \tag{4.1}$$

Exact measurement of gradient vector $\nabla \mathbf{J}(n)$ at time n requires prior knowledge of both the correlation matrix $\mathbf{R}$ of the input signals and the cross correlation vector $\mathbf{p}$ between the input signals and the desired signal. In practice, finding the correlation matrix $\mathbf{R}$ and the cross correlation vector $\mathbf{p}$ is difficult since it requires large number of samples. To overcome above problem, one solution is to use some estimates of $\mathbf{R}$ and $\mathbf{p}$ in gradient vector equation given by

$$\nabla \mathbf{J}(n) = -2\mathbf{p} + 2\mathbf{R}\mathbf{w}(n) \tag{4.2}$$

In LMS adaptive filter instantaneous estimates of $\mathbf{R}$ and $\mathbf{p}$ are used. Instantaneous values of $\mathbf{R}$ and $\mathbf{p}$ are defined as

$$\hat{\mathbf{R}} = \mathbf{x}(n)\,\mathbf{x}^H(n) \tag{4.3}$$

and

$$\hat{\mathbf{p}} = \mathbf{x}(n)\,d^*(n)\,. \tag{4.4}$$

Substituting equation 4.3, 4.4 in equation 4.2, then the estimate of gradient vector is given by

$$\nabla \hat{\mathbf{J}}(n) = -2\mathbf{x}(n)\,d^*(n) + 2\mathbf{x}(n)\,\mathbf{x}^H(n)\,\hat{\mathbf{w}}(n) \tag{4.5}$$

where $x(n)$ is input vector, $d(n)$ is desired response, e(n) is error signal, and $w(n) = [w_1 n, \ldots, w_M(n)]$ is tap weight vector of length M. Substituting the instantaneous estimate of gradient vector from equation 4.5 into steepest descent algorithm described in equation 2.22.

$$\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) - \mu\mathbf{x}(n)\left[d^*(n) - \mathbf{x}^H(n)\,\hat{\mathbf{w}}(n)\right] \tag{4.6}$$

So from above results we can define the LMS algorithm, which consists of following steps

a. **Compute filter output:**

$$y(n) = \hat{\mathbf{w}}^H(n)\,\mathbf{x}(n) \tag{4.7}$$

b. **Compute error:**

$$e(n) = d(n) - y(n) \tag{4.8}$$

c. **Update filter coefficient vector or tap-weight vector:**

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n) - \mu\hat{\mathbf{x}}(n)\,e^*(n) \tag{4.9}$$

where μ is called stepsize, which controls the convergence of the algorithm towards optimum solution (Wiener Hopf solution). The value of μ ranges from

$$0 < \mu < \frac{2}{MS_{max}}$$

, where M is filter length and S_{max} is maximum value of power spectral density of input signal $\mathbf{x}(n)$

4.1.2 Normalized Least Mean Square Adaptive filter

In LMS adaptive filter, coefficients are updated after every iteration which involves multiplication of three terms:

- Stepsize μ, which can be control.
- input vector $\mathbf{x}(n)$.
- conjugate of error signal $e(n)$ calculated at n^{th} iteration .

Update equation for tap-weight vector is given as

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n) + \mu\mathbf{x}(n)\,e^*(n) \tag{4.10}$$

From above equation, we see that adjustment of filter coefficients is directly proportional to filter input vector $\mathbf{x}(n)$. When $\mathbf{x}(n)$ is large LMS filter may suffer from gradient noise amplification factor. This difficulty can be solved if adjustment to filter coefficients at $n+1$ interval is normalized with respect to squared euclidean norm of the filter input vector $\mathbf{x}(n)$. This kind of filters are called as normalized least Mean Squares (NLMS) adaptive filter. Filter coefficients are updated by below equation

$$\hat{\mathbf{w}}(n) = \hat{(\mathbf{w})}(n) + \frac{\mu}{\epsilon + \|\mathbf{x}(n)\|^2}\mathbf{x}(n)\, e^*(n). \tag{4.11}$$

From above equation 4.11, it is clear that NLMS filter can be consider as LMS filter with variable stepsize, hence NLMS filter has faster convergence than LMS filter.

4.2 Two-Dimensional Adaptive Channel Estimation

Two dimensional (2D) channel estimation in OFDM systems uses frequency-domain and time-domain correlation of the channel. 2D MMSE channel estimator for OFDM systems is optimum, but it requires the knowledge of second order statistics of the channel, which in most of the cases are not available. Solution to above problem is adaptive channel estimation based on two dimensional LMS adaptive filter, which does not require any knowledge of channel statistics, and at the same time uses frequency-domain and time-domain correlation of the channel. Consider an OFDM system in which number of sub-carriers are N. The length of cyclic prefix is assumed to be larger than the length of the channel impulse response so that ISI and ICI are removed, thus the system can be represented as a set of parallel Gaussian channels and received signal can be represented as

$$Y[n,k] = H[n,k]\, X[n,k] + W[n,k] \tag{4.12}$$

where

$Y[n,k]$ is received OFDM symbol at n^{th} time and k^{th} subcarrier.

$X[n,k]$ is transmitted OFDM symbol at n^{th} time and k^{th} subcarrier.

$W[n,k]$ is additive complex Gaussian noise with zero mean, σ^2 variance , and uncorrelated for different value n and k, where $k = 1, 2, \ldots, N$.

4.2.1 Adaptive two-dimensional LMS channel estimation for OFDM systems

Two dimensional LMS channel estimator for OFDM systems takes the advantage of correlation in both time-domain and frequency-domain [46], [47]. No pilot symbols are used in adaptive channel estimator for OFDM systems. Past channel responses are used to estimate channel response for current OFDM symbol. Past L channel attenuation vector in frequency domain are arranged in a column vector $\mathbf{p}$ of order $NL \times 1$, then estimate of channel attenuation vector $\mathbf{h}$ (or channel response in frequency-domain) for current OFDM symbol is given by

$$\hat{\mathbf{h}} = \mathbf{G}\mathbf{p} \tag{4.13}$$

where $\mathbf{G}$ is called as weight coefficient matrix of order $N \times NL$ represented as

$$\mathbf{G} = \begin{bmatrix} G_{11} & \ldots & G_{1(NL)} \\ \vdots & \ddots & \vdots \\ G_{N1} & \ldots & G_{N(NL)} \end{bmatrix}$$

and $\hat{\mathbf{h}}$ is estimate of the channel attenuation vector of order $N \times 1$ and is represented as

$$\hat{\mathbf{h}} = \mathbf{G}\mathbf{p}.$$

Also, $\mathbf{h}$ is actual channel attenuation vector and represented as

$$\mathbf{h} = \begin{bmatrix} H[n,1] \\ \vdots \\ H[n,N] \end{bmatrix}$$

where $H[n,k]$ represents attenuation of the channel for n^{th} symbol at k^{th} sub-carrier and are arranged in vector $\mathbf{p}$ as

$$\mathbf{p} = \begin{bmatrix} H[n-1,1] \\ \vdots \\ H[n-1,N] \\ \vdots \\ H[n-L,1] \\ \vdots \\ H[n-L,N] \end{bmatrix}$$

Estimate of the channel attenuation vector $\hat{\mathbf{h}}$ updates its value using algorithm given below

a. **Filtering:**

$$\hat{\mathbf{h}}(n) = \mathbf{G}(n)\mathbf{p}(n)$$

b. **Computing Error:**

$$\mathbf{e}(n) = \mathbf{h}(n) - \hat{\mathbf{h}}(n)$$

or

$$\hat{\mathbf{e}}(n) = \mathbf{h}(n) - \mathbf{G}(n)\mathbf{p}(n)$$

c. **Updating the weight coefficient matrix:**

$$\mathbf{G}(n+1) = \mathbf{G}(n) + \mu \mathbf{e}(n)\mathbf{p}^H(n)$$

where μ is positive stepsize, whose maximum and minimum value is given by

$$0 < \mu < \frac{2}{MS_{max}} \tag{4.14}$$

where S_{max} is maximum value of the power spectral density of input signal and M is order of adaptive filter. Initial value for $\mathbf{G}$ is assumed to be 1.

4.2.2 Two-Dimensional Normalized Least Mean Square Algorithm

Two-Dimensional LMS adaptive channel estimator has disadvantage that the coefficient matrix $\mathbf{G}$ adjust its value proportional to vector $\mathbf{p}(n)$. However, when $\mathbf{p}(n)$ is large gradient noise amplification problem may arise. To solve this problem two-dimensional normalized least mean square (NLMS) adaptive filter is used, which involves following steps to update its coefficients

a. **Filtering:**

$$\hat{\mathbf{h}}(n) = \mathbf{G}(n)\mathbf{p}(n)$$

b. **Computing Error:**

$$\mathbf{e}(n) = \mathbf{h}(n) - \hat{\mathbf{h}}(n)$$

or

$$\mathbf{e}(n) = \mathbf{h}(n) - \mathbf{G}(n)\mathbf{p}(n)$$

c. **Updating the weight coefficient matrix:**

$$\mathbf{G}(n+1) = \mathbf{G}(n) + \frac{\mu}{\|\mathbf{p}(n)\|^2}\mathbf{e}(n)\mathbf{p}^H(n)$$

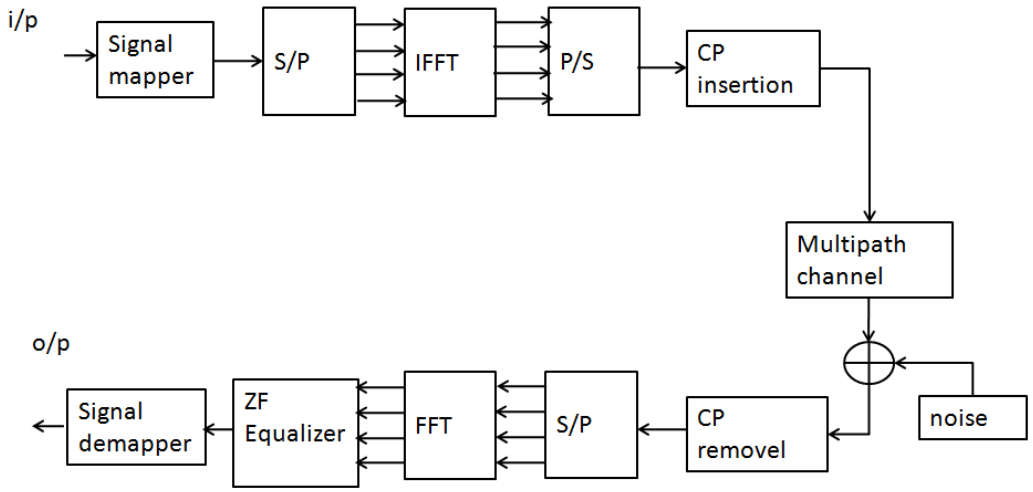

Figure 4.2: Block diagram for modified NLMS channel estimation

Hence problem of gradient noise amplification can be solved. It can be seen that NLMS adaptive channel estimator is a special case of LMS adaptive channel estimator since overall stepsize is varying according to input vector.

4.3 Proposed Method

LMS adaptive estimator and NLMS adaptive estimator used for channel estimation have slow convergence [48], hence large number of OFDM symbols are required to converge towards optimum solution (Wiener Hopf equation). Convergence of NLMS adaptive channel estimator can be increased with the use of variable stepsize.

4.3.1 Adaptive two-Dimension NLMS channel estimator for OFDM with variable stepsize

Convergence towards optimum solution can be increased by using variable stepsize, in above discussed methods stepsize μ was fixed. Stepsize can be varied after arrival of each OFDM symbol. Convergence of NLMS algorithm can be increased by using variable stepsize. In [49], modified NLMS algorithm with variable stepsize is proposed. The convergence of Channel estimator based on 2D-NLMS [35] can be increase, if instead of fixed stepsize variable stepsize is used. The 2D- adaptive channel estimator for OFDM system is proposed, which is based on NLMS algorithm with variable stepsize [49]. Update equation for $G(n)$ is given by

$$\mathbf{G}(n+1) = \mathbf{G}(n) + \frac{\mu}{\|\mathbf{p}(n)\|^2}\mathbf{e}(n)\mathbf{p}^H n \tag{4.15}$$

In above equation if μ is a variable, then convergence performance of 2D-NLMS algorithm can be improved. Stepsize can be vary by using below equation

$$\mu(n+1) = \frac{\sum_{i=0}^{n} \alpha^i \mathbf{e}^2(n-i)}{\sum_{j=0}^{n} \beta_j \mathbf{e}^2(n-j)} \tag{4.16}$$

where $0 < \alpha < \beta < 1$, $\mu(n) \in [\mu_{min}, \mu_{max}]$ and $\mathbf{e}^2(n)$ is energy of the instantaneous output error signal. From above equation it is clear that numerator is always less than denominator and as number of iteration increases $\mu(n)$ decreases.

Also from above equation it can be shown that

$$\sum_{i=0}^{n} \alpha_i \mathbf{e}^2(n-i) \leq \sum_{j=0}^{n} \beta_j \mathbf{e}^2(n-j)$$

Variable stepsize provides large value at the starting of adaptation process, and as number of iteration increases value of stepsize decreases accordingly. Above stepsize has one more advantage, when very rapid change occurs in incoming data then stepsize increases its value accordingly.

Suppose

$$\epsilon_i(n) = \sum_{i=0}^{n} \alpha^i \mathbf{e}^2(n-i), \tag{4.17}$$

after rearranging above equation, $\epsilon_i(n)$ can be rewrite as

$$\epsilon_i(n) = \sum_{i=0}^{n} \alpha^{n-i} \mathbf{e}^2(i) \tag{4.18}$$

$$= \alpha \sum_{i=0}^{n-1} \alpha^{n-i-1} \mathbf{e}^2(i) + \mathbf{e}^2(n)$$

$$\alpha\epsilon_i(n-1) + \mathbf{e}^2(n)$$

hence

$$\epsilon_i(n) = \alpha\epsilon_i(n-1) + \mathbf{e}^2(n) \tag{4.19}$$

similarly

$$\epsilon j(n) = \beta\epsilon_j(n-1) + \mathbf{e}^2(n) \tag{4.20}$$

$$\mu(n) = \frac{\epsilon_i(n)}{\epsilon_j(n)} \tag{4.21}$$

Sufficient condition to guarantee the convergence of above modified NLMS algorithm is given by [50]

$$0 < E\{\mu_n\} < \frac{2}{\lambda_x} \tag{4.22}$$

So from above discussion modified 2D-NLMS channel estimator with variable stepsize can be defined as

a. **Filtering:**

$$\hat{\mathbf{h}}(n) = \mathbf{G}(n)\mathbf{p}(n)$$

b. **Computing Error:**

$$\hat{\mathbf{e}}(n) = \mathbf{h}(n) - \mathbf{h}(n)$$

or

$$\hat{\mathbf{e}}(n) = \mathbf{h}(n) - \mathbf{G}(n)\mathbf{p}(n)$$

c. **Updating the weight coefficient matrix.**

$$\mathbf{G}(n+1) = \mathbf{G}(n) + \frac{\mu(n)}{\|\mathbf{p}(n)\|^2}\mathbf{e}(n)\mathbf{p}^H(n)$$

$\mu(n)$ is update by

$$\mu(n) = \frac{\epsilon_i(n)}{\epsilon_j(n)}$$

where $\epsilon_i(n)$ and $\epsilon_j(n)$ are updated by equation 4.19 and 4.20

4.4 Simulation results and comparison

Parameters used for simulation are given in table 4.4. μ_{max} and μ_{min} depends on the eigen value spread of signal, in simulation value of $\mu_{max} = .0881$ and $\mu_{min} = 3 \times 10^{-4}$ is used.

Parameter	**value**
Total Bandwidth	20 MHz
Carrier frequency F_c	5 GHz
CP time duration T_{CP}	3.2 μs
OFDM symbol time T	12.8μs
Total number of sub-carrier N_c	256
Doppler spread f_d	100Hz
FFT size	256
Symbol interval T_s	16μs
sub-carrier spacing F	0.03125 MHz
Data symbol constellation	QPSK

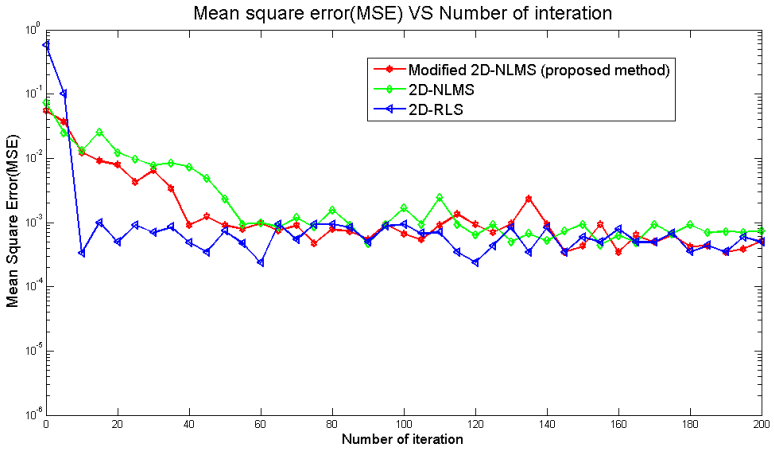

Figure 4.3: Plot between mean square error (MSE) and number of iteration

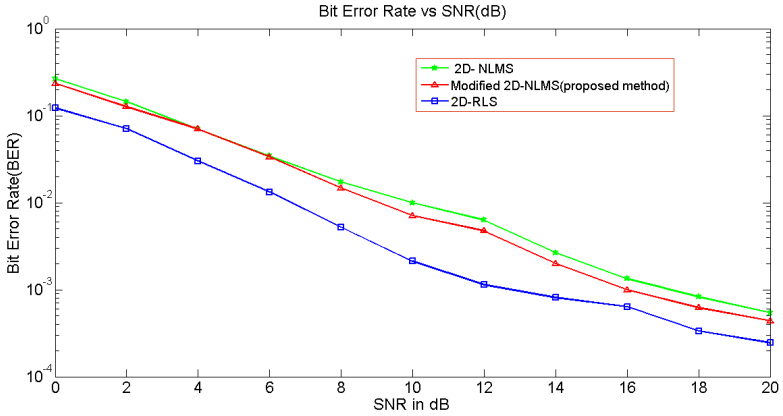

Figure 4.4: Plot between bit error rate (BER) and SNR in dB

Channel is assumed to be slow fading channel, and value of L is chosen to be $L = 5$. The value of $\alpha = 0.97$ and $\beta = 0.99$ are used since these value provides good convergence [49].

Simulation results of proposed method (2D- NLMS adaptive channel estimator with variable stepsize) is compared with 2D NLMS adaptive channel estimator [35] and 2D-RLS adaptive channel estimator [36]. Figure 4.3 shows performance of channel estimator in terms of MSE and figure 4.4 shows the performance in terms in bit error rate (BER). Simulation results show that proposed method has better convergence as compared to 2D-NLMS algorithm. 2D- RLS algorithm has the best performance in terms of convergence amongst all we discussed since it converge to exact solution, but have high computational complexity.

Chapter 5

Conclusion and future work

5.1 Conclusion

From 1960 to today, OFDM is a strong transmission technique to overcome channel effects in wireless environment. Although OFDM gives many advantages, it suffers from many complex problems. Channel estimation can be considered an important part of OFDM receiver since the wireless channel is unpredictable and changes continuously with time, hence the knowledge of the channel can reduce the complexity of equalizer in order to compensate for the channel distortion.

In this thesis various pilot symbols based channel estimation techniques are studied and their results are compared with proposed method, which is based on regularized least squared estimation technique. Regularized least squared estimation technique performs well under low SNR as compared to LS, but the price paid is in terms of in computation time.

Adaptive channel estimation techniques are also studied, These techniques does not require any pilot symbols or any statistical knowledge of channel. Results of 2D-NLMS and the proposed modified 2D NLMS adaptive filter with variable stepsize channel estimator are compared. Proposed method provides good convergence as compared to 2D NLMS method and also the proposed method is more robust against

any rapid change in signal or channel.

5.2 Future work

Regularized least squared method is a powerful method for channel estimation, since we have the freedom to use constraints. In this thesis prior distribution of channel is used as a constraint, but there are various channel parameters which can also be used as a constraint. Power delay profile, delay spread of channel, or more precisely any information available can be used as a constraint to get better performance.
Adaptive channel estimators are more effective as they do not require channel statistics, but they have high computational complexity. Designing less complex adaptive channel estimators can be an interesting work for future.

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